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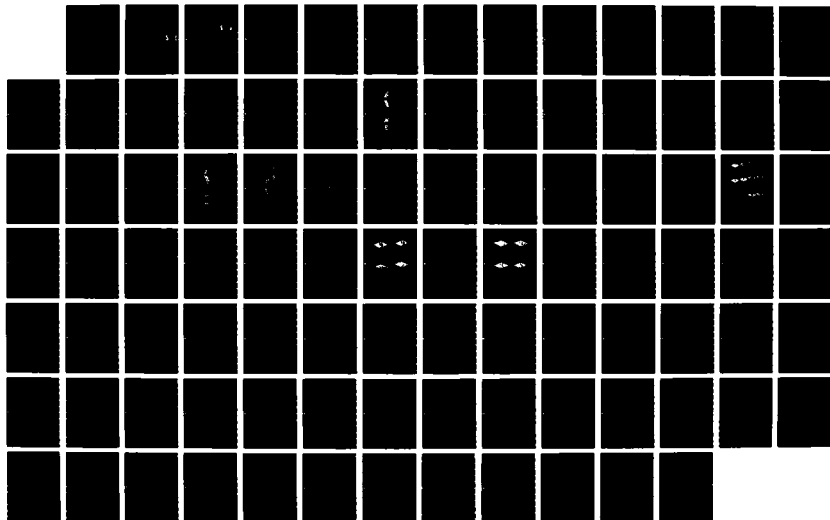
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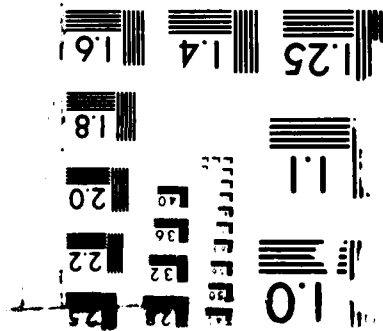
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FOR MULTI-APERTURE OPTICAL SYSTEMS

THESIS

Steven M. Watson  
Major, USAF

AFIT/GEP/ENP/87M-1

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FOR MULTI-APERTURE OPTICAL SYSTEMS

THESIS

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology  
Air University  
In Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science in Engineering Physics

Steven M. Watson, B.S., M.S.  
Major, USAF

March 1987



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## Preface

Large sized optics will be required to satisfy the Strategic Defense Initiative surveillance needs. Because of size restrictions for transportation into space, multi-aperture systems could be utilized. The diffraction patterns for the multi-aperture systems are unique. As a result, a two-point resolution criterion needs to be established and is presented in this paper. This threshold criterion applies to any multi-aperture system.

I would like to thank my advisor and good friend, Major James Mills, for his patience, guidance, and his genuine concern. He gave me the opportunity to continue on and build my confidence. I would also like to thank his wife, Patricia, for her friendship and invaluable assistance with my computer programs. Finally, I am indebted to Wanda Kucharski for her understanding and support. She helped me through the most demanding times.

Steven M. Watson

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## ABSTRACT

Two-point resolution criteria is the classic way of comparing telescopes. However, the standard two-point resolution criterion is not appropriate for multi-aperture systems. This paper proposes a new two-point resolution criterion based on the idea of thresholding the irradiances of the resulting far-field diffraction patterns of multi-aperture optical systems. The threshold was defined as a fraction of the central lobe irradiance. The thresholds varied from 0.1 to 0.9 of the central lobe irradiance of the far-field diffraction patterns. Theoretical data of normalized irradiance versus point separation for various multi-aperture optical systems were presented. The two-point resolution for these configurations was analyzed. The two-point resolution criterion using thresholds was demonstrated. The threshold criterion provided the information necessary to compare the two-point resolution performance of a particular multi-aperture optical system illuminated coherently and incoherently. Also, this criterion allowed the comparison of the two-point resolution performance of systems composed of three, four, and six subapertures illuminated incoherently.



## TWO-POINT RESOLUTION CRITERION FOR MULTI-APERTURE OPTICAL SYSTEMS

### I. Introduction

The resolution necessary to satisfy the Strategic Defense Initiative (SDI) surveillance needs requires large diameter optical systems. The size required exceeds the size limits for transportation into appropriate orbits around the earth. Multi-aperture optical systems are one of the many proposed solutions for this problem. A multi-aperture system is composed of several identical subapertures arranged such that the effective diameter of the system is greater than that of a single subaperture. Figure 1.1 depicts such an arrangement (b) compared with an equivalent size large optic (a). A multi-aperture system has many advantages over a single aperture system. However, there are also disadvantages since the area contained within the effective diameter of any multi-aperture system is not completely filled. The possible number of configurations of the apertures is large. Thus the imaging properties of the various configurations need to be investigated and compared. The two-point resolution criterion is the classic way of comparing single aperture telescopic

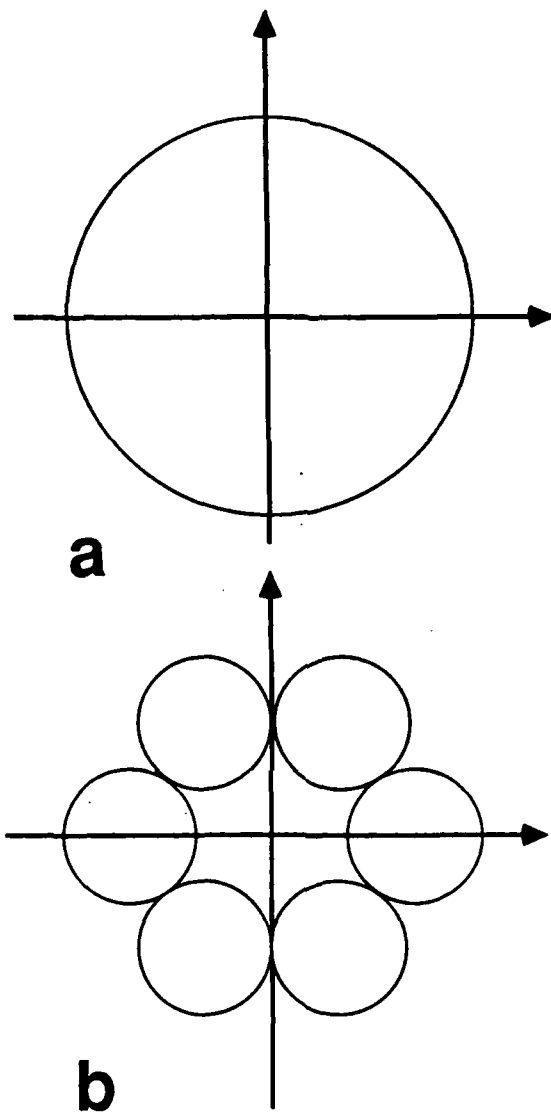


Fig. 1.1. a) Single Large Optic and b) Multi-Aperture System with Equivalent Diameter.

systems. However, there is no standard two-point resolution criterion for multi-aperture systems. The purpose of this research was to establish a two-point resolution criterion for multi-aperture systems.

The investigation of the two-point resolution problem involved a computer analysis of symmetrically arranged three, four, and six aperture systems composed of identical circular apertures. Refer to Figure 1.2 for the general configuration of these systems. Since the area inside these multi-aperture systems is not filled, the impulse response compared with that of a single large optic is considerably different. Figure 1.3 illustrates this with a comparison of the impulse response of a single large optic with that of a six aperture system of an equivalent diameter (Many of the plots in this paper have an illustration of the multi-aperture system, with a depiction of the relative spacing of the subapertures, superimposed in the upper right-hand corner). The U and V axis labels represent the normalized distance in the observation plane. The diameters of the two optical systems are considered equivalent when the diameter of the single large optic equals the diameter of a circle which just encloses the multi-aperture system. The impulse response of the six aperture system is characterized by a central lobe, the central peak of an irradiance pattern of an imaged point source, surrounded by side lobes. Depending on the configuration of the multi-aperture system, the irradiance of these side lobes can approach the irradiance of the central lobe. When this occurs, the standard two-point resolution

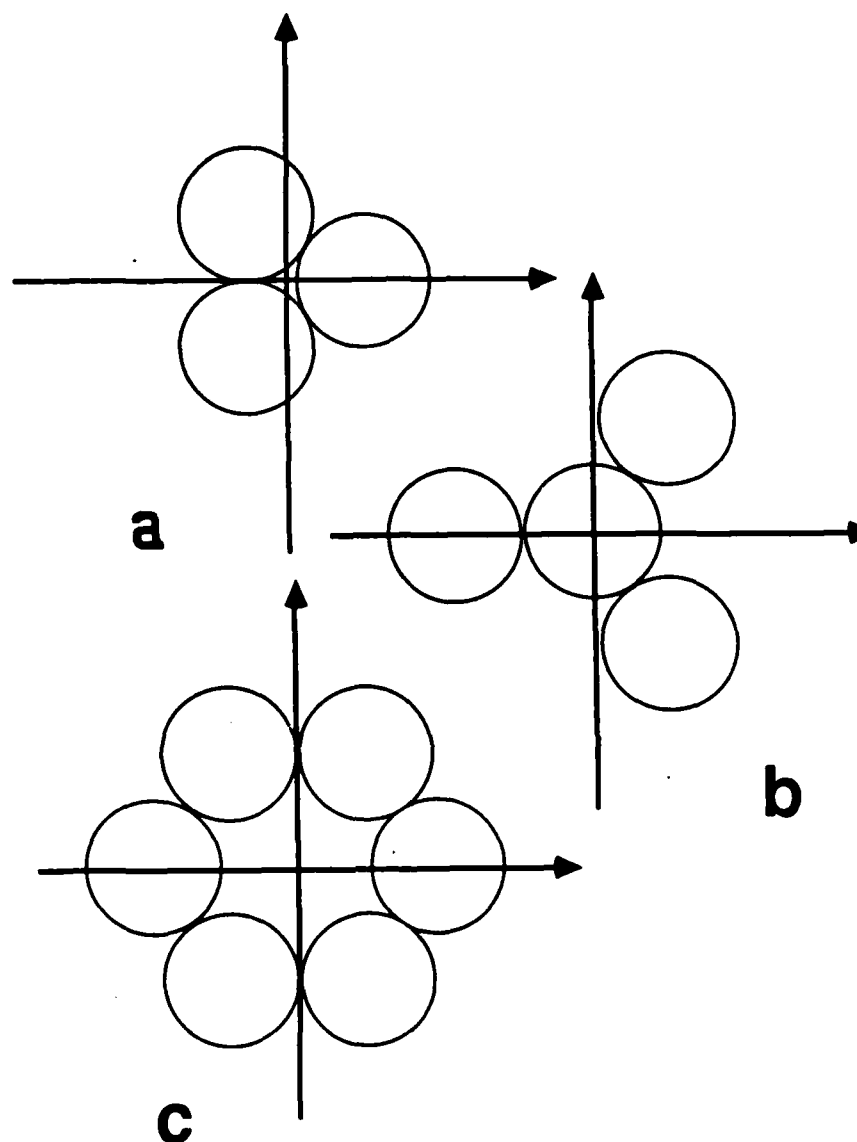


Fig. 1.2. Depiction of Symmetric a) Three, b) Four, and c) Six Aperture Systems.

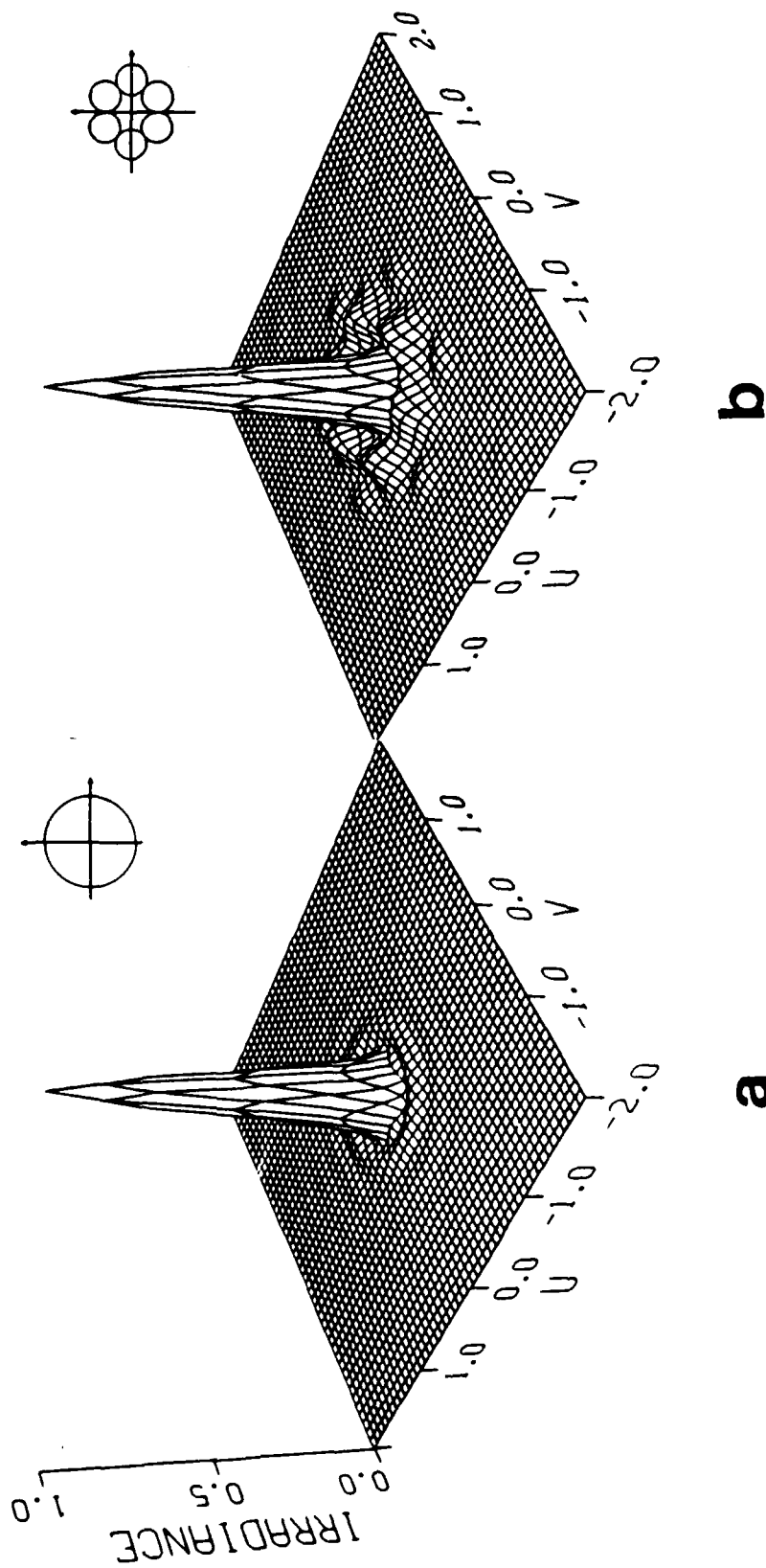


Fig. 1.3. Impulse Response of a) Single Large Optic and  
b) Six Aperture System of Equivalent Diameter.  
The Aperture System is Depicted in the Upper  
Right-Hand Corner of each Plot.

criterion for single aperture systems is not appropriate.

The initial portion of the research involved the derivation of an expression describing the impulse response of any multi-aperture system. This expression was used to find the far-field diffraction irradiance patterns of the multi-aperture systems illuminated coherently and incoherently by two closely-spaced point sources. The distance between the two point sources was varied as well as the distance between each aperture and the origin of the multi-aperture systems. The effect on the far-field diffraction patterns was noted and used to establish a two-point resolution criterion for multi-aperture systems.

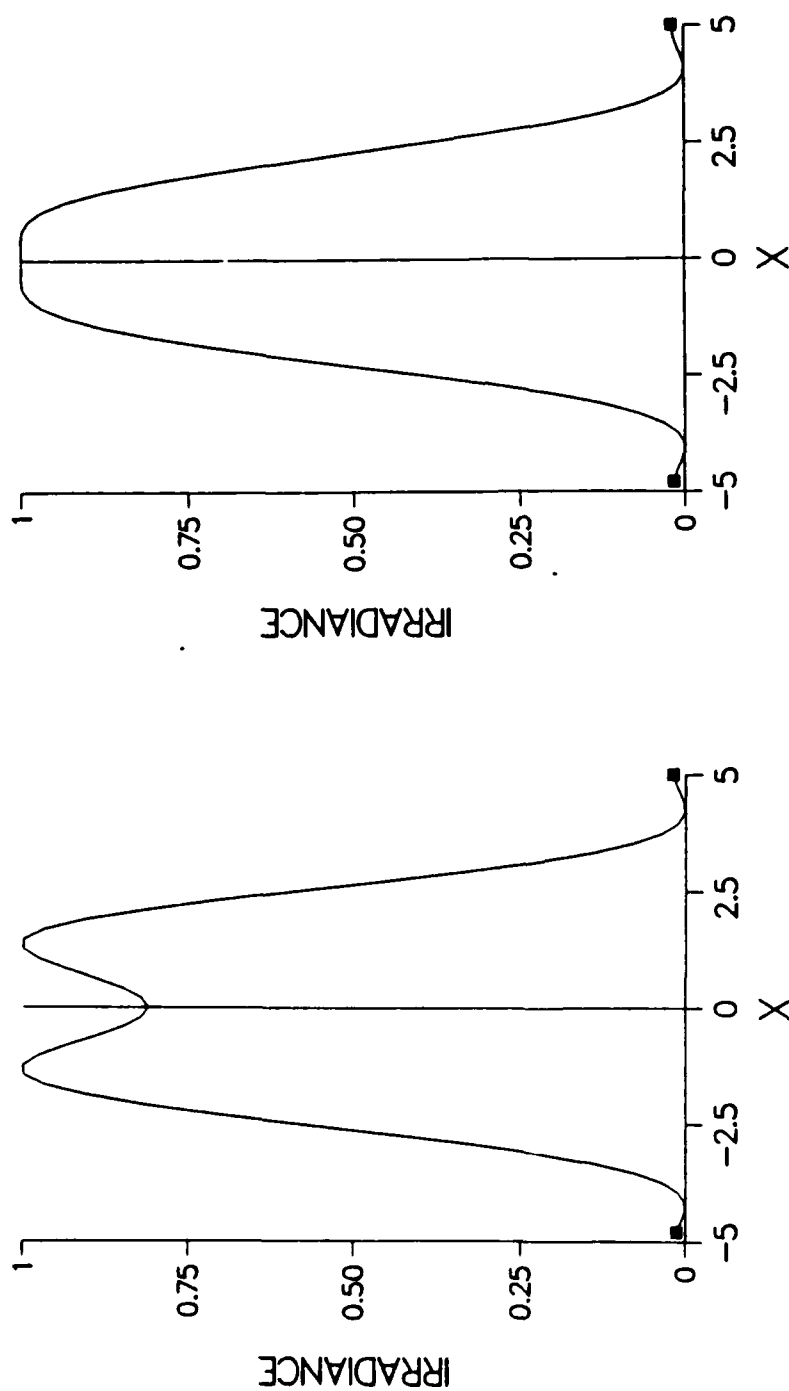
### Background

Fender (1) discussed the advantages and disadvantages of synthetic (multi) aperture systems compared to single aperture systems. She examined the far field irradiance patterns of multi-aperture systems and derived an expression for the impulse response of a symmetric hex-aperture system.

The idea of two-point resolution for single aperture systems has been examined by Lord Rayleigh (2) and Sparrow (3). The Rayleigh criterion states that two mutually incoherent point sources are just resolved when the center of the Airy disk produced by one point source falls on the first minimum of the Airy disk produced by the second point source (4:130).

This results in approximately a 19 percent dip in the center of the pattern as compared to the maximum value. Figure 1.4a depicts a cross-section of the resulting irradiance pattern when this condition is met.

A second criterion which is commonly used is the Sparrow criterion (8:21). The irradiance pattern of 2 points that are resolved is characterized by two diffraction maxima with a central minimum between them. As the two points are brought closer together, the central minimum approaches the value of the adjacent maxima until it just disappears. Sparrow referred to this point as the "undulation condition." At this point, the two point sources are considered to be just resolved by the Sparrow criterion (5:19). Figure 1.4b depicts the Sparrow criterion for two point sources.



**a**

**b**

Fig. 1.4 a) Rayleigh and b) Sparrow Resolution Criterion.



## II. Theory

One measure of the performance of an optical system is the ability to resolve two point sources (8:20). The goal of the present research was to establish a two-point resolution criterion for multi-aperture optical systems. In order to accomplish this, it was necessary to model and analyze the far-field diffraction patterns produced by any multi-aperture optical system illuminated by two closely-spaced point sources. To form a baseline for this theoretical analysis, the generalized form of the impulse response for any multi-aperture system was derived. The next step involved the derivation of the complex field amplitude for any multi-aperture system illuminated by two closely-spaced point sources. Finally, this expression for the complex field amplitude was used to solve for the irradiance, at the image plane, for coherent and incoherent illumination. Implicit in the following analysis was that the complex field amplitudes of the two point sources were equal.

## MULTI-APERTURE IMPULSE RESPONSE

Figure 2.1 depicts the configuration used to model a multi-aperture system (4:91). In this analysis, the field produced by a single point source is propagated from the object plane, through the aperture plane, to the image plane.

The subapertures which comprise the multi-aperture systems are identical circular apertures. Figure 2.2 illustrates one form of a multi-aperture system. This particular arrangement of six apertures illustrates the variables used in the calculation of the impulse response for all of the multi-aperture systems. In this analysis "a" was the radius of each of the subapertures; n was equal to the number of apertures in the system;  $x_n$  and  $y_n$  described the location of the centers of the  $n^{\text{th}}$  subaperture;  $\theta_n$  was the angle in degrees from the x axis; and  $p_n$  was the distance of the  $n^{\text{th}}$  subaperture from the origin of the system and was expressed in terms of multiples of the subaperture radius. Figure 2.3 illustrates two six aperture systems with aperture-origin separations,  $p_n$ , equal to 2.00a and 3.00a.

The pupil function of a multi-aperture function can be expressed as the convolution of one of the apertures with the delta functions which describe the location of the centers of each aperture. As a result, the generalized pupil function can be written as

$$P(x,y) = \text{circ}[r/a] * \sum_{n=1}^n \delta(x - p_n \cos \theta_n, y - p_n \sin \theta_n) \quad (2.1)$$

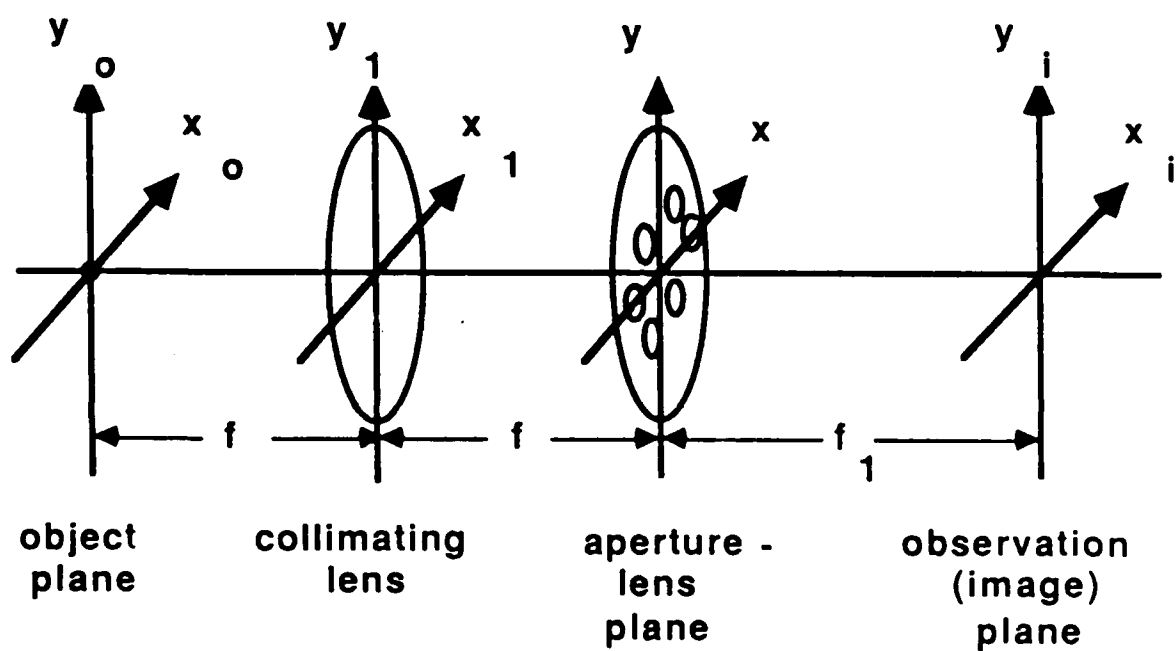


Fig. 2.1. Configuration for Observing Impulse Response for a Multi-Aperture System.

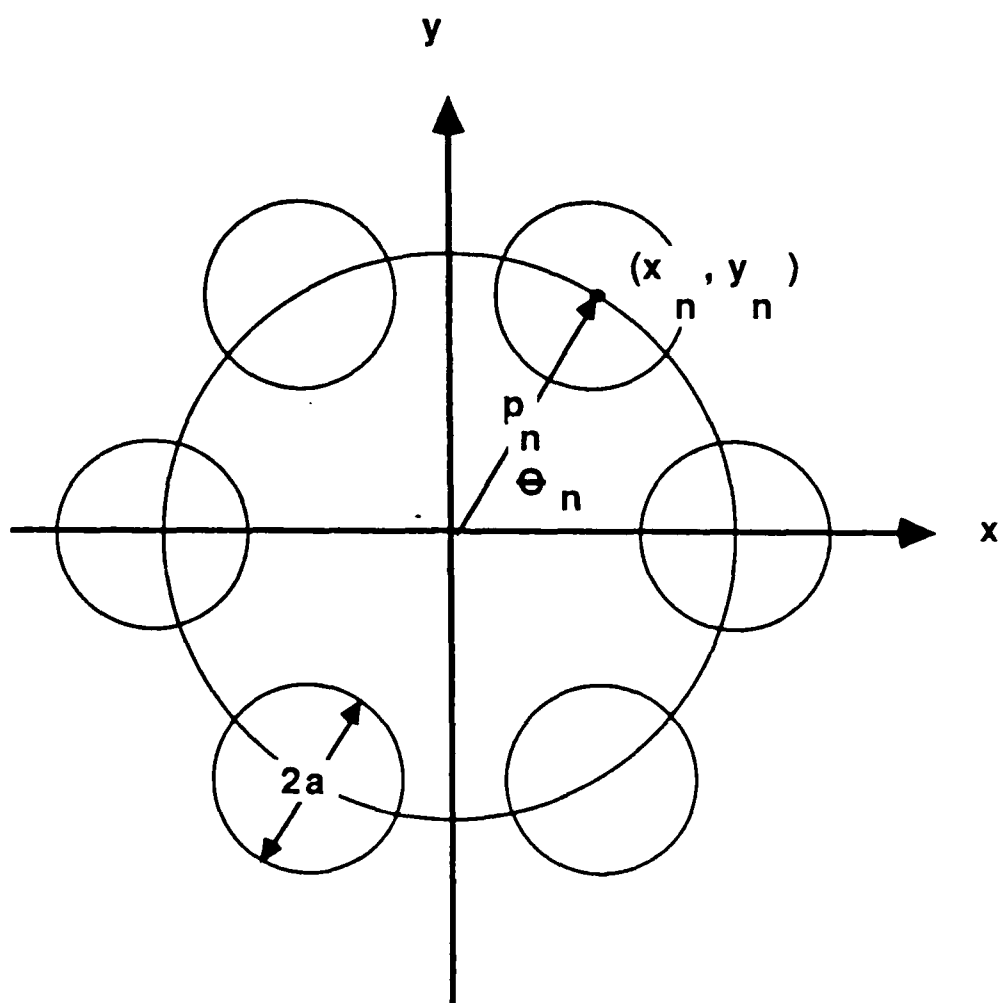


Fig. 2.2. Example of a Multi-Aperture System with an Aperture-Origin Separation of  $p_n$  and Subaperture Radius of  $a$ .

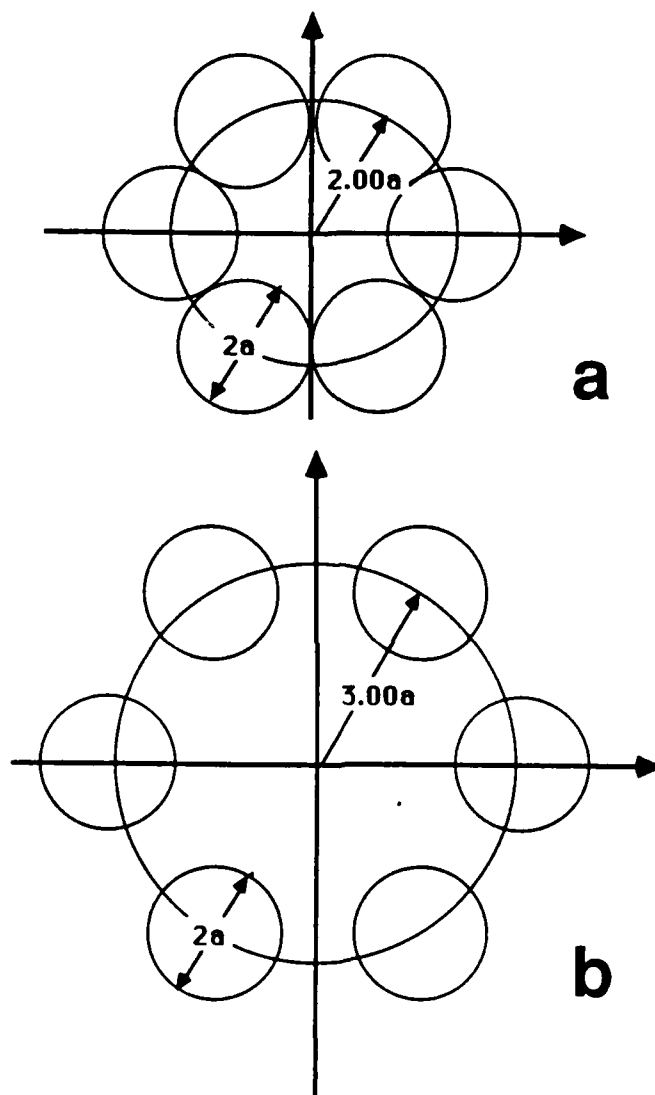


Fig. 2.3. Six Aperture Systems with Aperture-Origin Separations,  $p_n$ , of a)  $2.00a$  and b)  $3.00a$ .

where  $\text{circ}[r/a] = 1$  if  $r/a$  is less than or equal to 1, otherwise, it equals 0. The circ function describes a single aperture of the multi-aperture system where  $a$  is the radius of the circular aperture and  $r=(x^2+y^2)^{1/2}$ .

The impulse response,  $h(x_1, y_1)$ , is defined to be the Fourier transform of the exit pupil (4:111). Therefore, the impulse response is

$$\begin{aligned} h(x_1, y_1) &= F\{P(x, y)\} \\ &= F\{\text{circ}[r/a]\} \cdot F\left\{\sum_1^n \delta(x - p_n \cos \theta_n, y - p_n \sin \theta_n)\right\} \\ &= a J_1 \left[ \frac{2\pi a \sqrt{(x_1/\lambda f)^2 + (y_1/\lambda f)^2}}{\sqrt{(x_1/\lambda f)^2 + (y_1/\lambda f)^2}} \right] \\ &\quad \cdot \sum_1^n \exp[-i2\pi(x_1 p_n \cos \theta_n + y_1 p_n \sin \theta_n)/\lambda f] \quad (2.2) \end{aligned}$$

where  $F\{ \}$  denotes the Fourier transform. Note that for each delta function, which describes the location of the center of a particular subaperture in the aperture plane, there corresponds a plane wave traveling from the subaperture to the focal point.

The envelope function of this impulse response is

$$\frac{a J_1 \left[ 2\pi a \sqrt{(x_1/\lambda f)^2 + (y_1/\lambda f)^2} \right]}{\sqrt{(x_1/\lambda f)^2 + (y_1/\lambda f)^2}} \quad (2.3)$$

which is a scaled version of the impulse response for a single subaperture of the multi-aperture system. This envelope function is modulated by cosine fringes that arise from the

complex exponentials in the remaining portion of equation (2.2). The frequency of the cosine fringes is a function of the aperture-origin spacing,  $p_n$ , of the individual subapertures from the origin.

To demonstrate this modulation, consider a six aperture system with an aperture-origin separation of  $2.00a$  as depicted in Figure 2.3a. Figure 2.4a shows the irradiance of the envelope function of equation (2.2). Figure 2.4b is a plot of the irradiance for the entire equation (2.2) and demonstrates how the envelope function has been modulated by the cosine fringes resulting in side lobes. When the apertures were moved further apart, the side lobes increased in amplitude. Figures 2.5a through c illustrate this phenomenon for a six aperture system with aperture-origin separations of  $2.00a$ ,  $3.00a$ , and  $4.00a$ , respectively. The growth of the side lobes make the standard single aperture two-point resolution criterion, Rayleigh and Sparrow criteria, inappropriate since these criteria do not account for the side lobes.

#### MULTI-APERTURE - TWO POINT SOURCE ILLUMINATION

The resulting far-field diffraction pattern due to the illumination of multi-aperture systems by two closely-spaced point sources were computed by propagating the fields from the sources through the configuration depicted in Figure 2.6.

The point sources were on the  $x_0$  axis and located distances  $+b$  and  $-b$  from the  $y_0$  axis. The field at the object

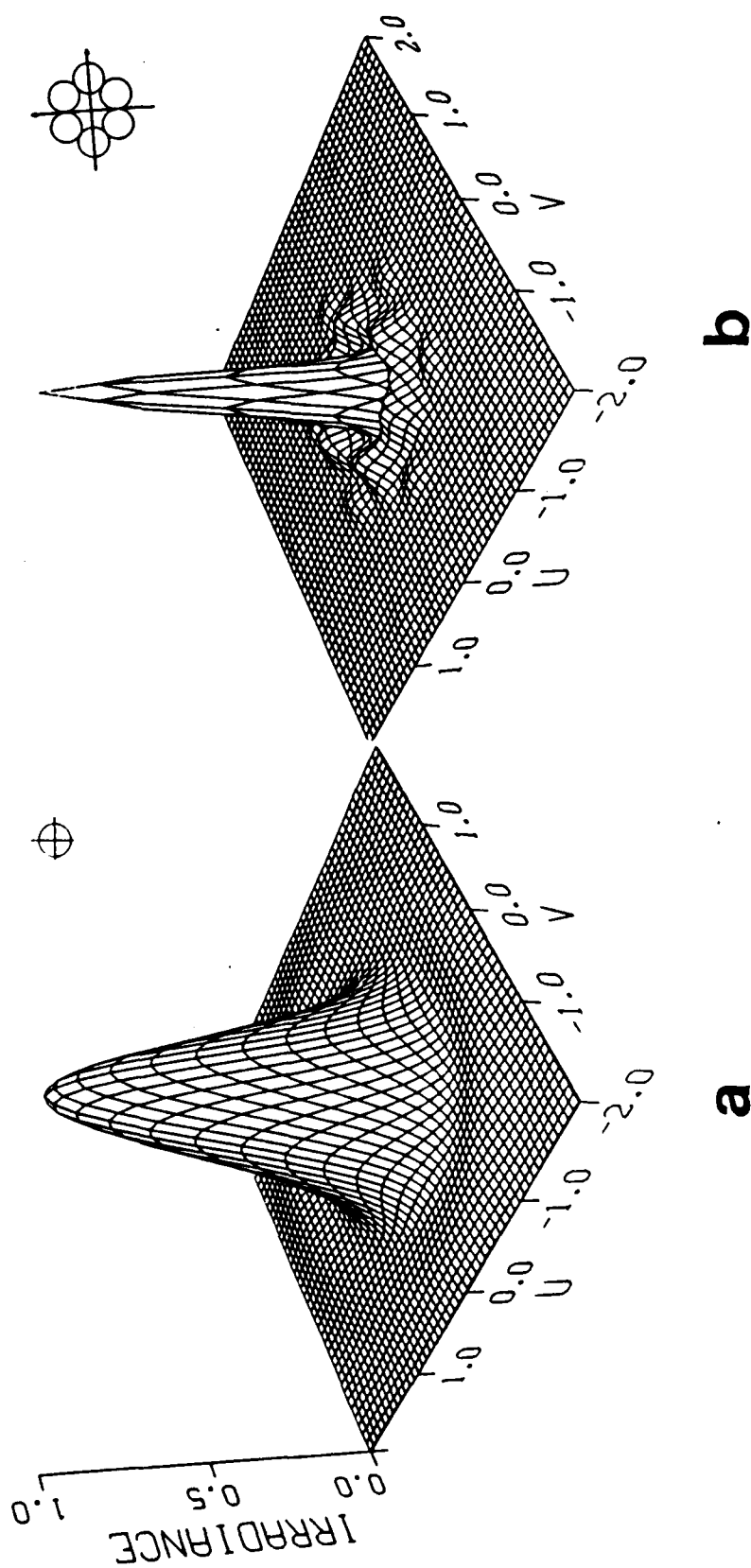


Fig. 2.4. Irradiance Pattern for a) Single Aperture of a Multi-Aperture System and b) Six Aperture System with Aperture-Origin Separation of 2.00a.



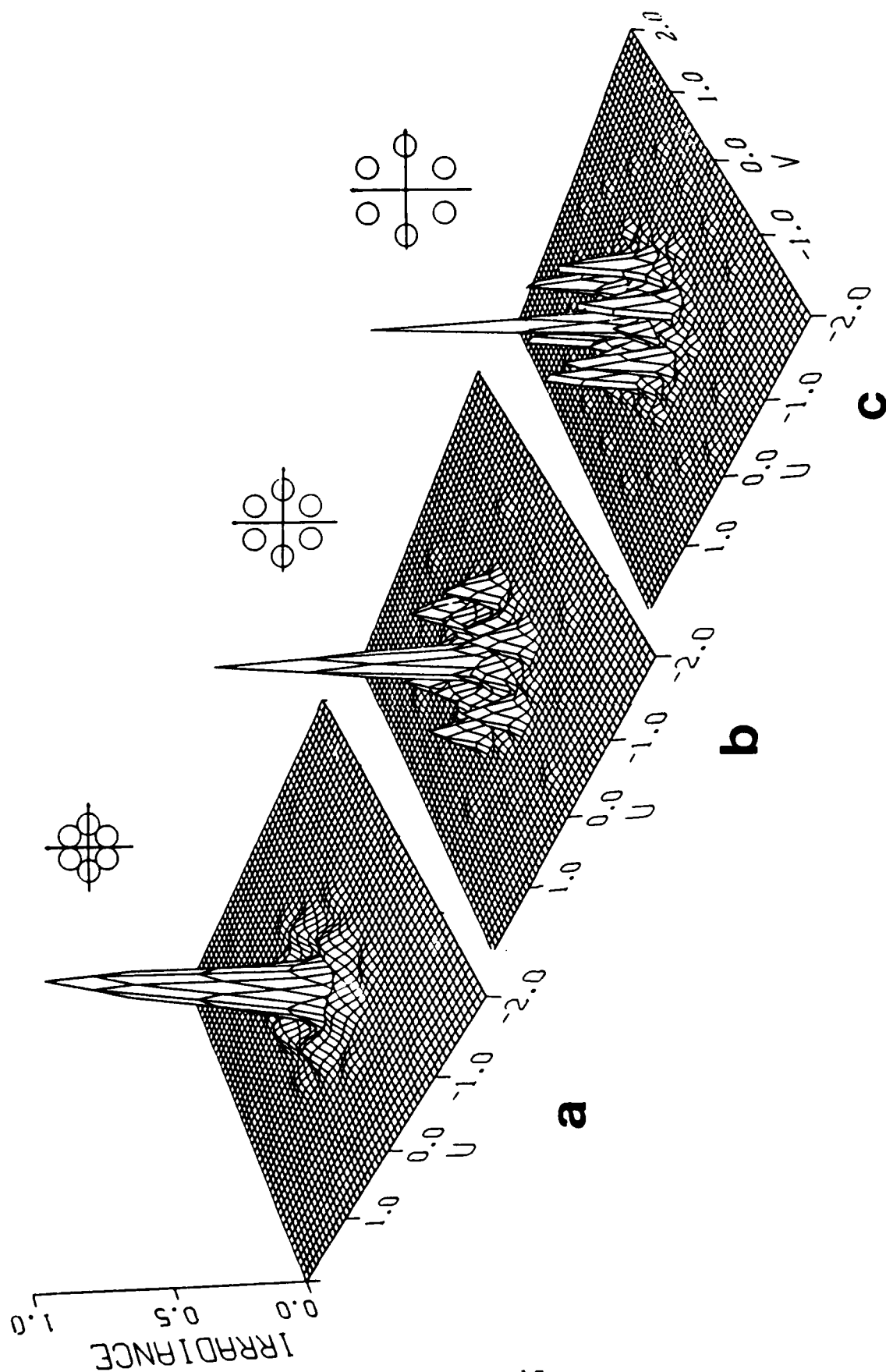


Fig. 2.5. Far-Field Irradiance Patterns for Six Aperture System with Aperture-Origin Separations of  
a) 2.00a, b) 3.00a, and c) 4.00a.

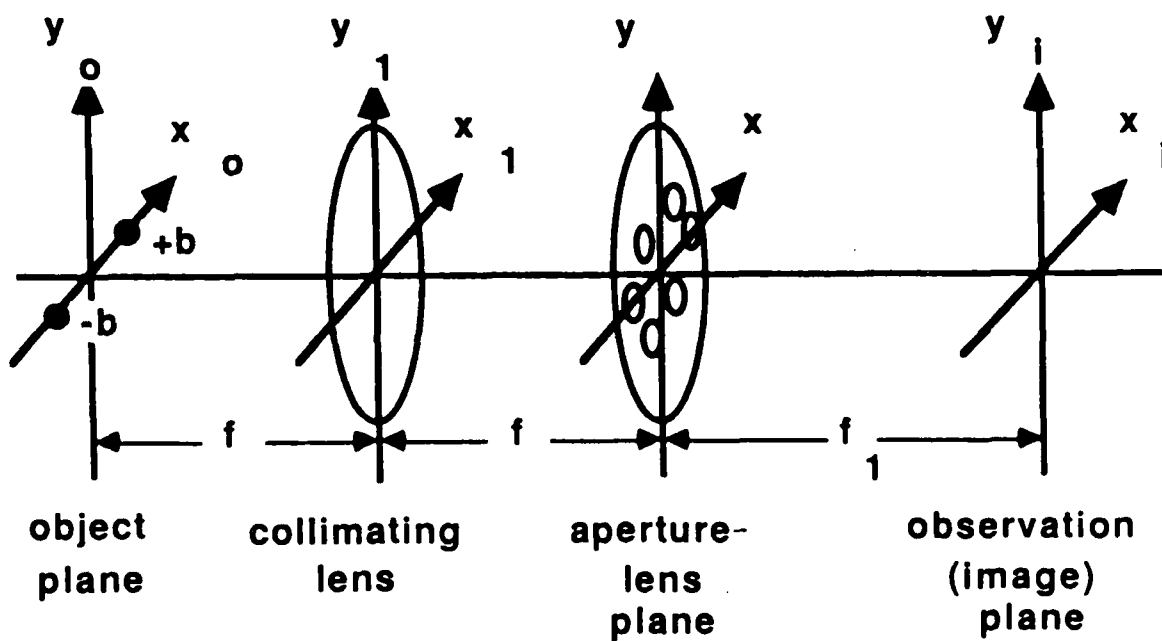


Fig. 2.6. Configuration for Observing Irradiance Patterns from Two Point Sources on  $X_0$  Axis.

plane was described by

$$u_0(x_0, y_0) = \delta(x_0 - b, y_0) + \delta(x_0 + b, y_0) \quad (2.4)$$

Propagating the fields from the two point sources through the configuration depicted in Figure 2.6 resulted in two cases depending on whether the sources were coherent or incoherent with respect to each other.

### Coherent Analysis

Propagation of the fields from two closely-spaced coherent point sources to the image plane resulted in the complex field amplitude

$$u(x_1, y_1) = \left\{ \begin{array}{l} \delta[(x_1 - b)/\lambda f, y_1/\lambda f] + \\ \delta[(x_1 + b)/\lambda f, y_1/\lambda f] \end{array} \right\} \cdot \left\{ \begin{array}{l} \left[ \frac{2a^2 J_1 \left[ 2\pi a \sqrt{(x_1/\lambda f)^2 + (y_1/\lambda f)^2} \right]}{2a \sqrt{(x_1/\lambda f)^2 + (y_1/\lambda f)^2}} \right] \\ \cdot \left[ \sum_1^n \exp[-i2\pi p_n (x_1 \cos \theta_n + y_1 \sin \theta_n)/\lambda f] \right] \end{array} \right\} \quad (2.5)$$

The coordinates in the image plane were normalized using the relations

$$U = x_1 d\pi/\lambda f \quad (2.6)$$

and

$$V = y_1 d\pi/\lambda f \quad (2.7)$$

Letting  $\lambda f = 1.0$  and performing the convolution resulted in

$$\begin{aligned}
u_1(U,V) = & \left\{ \frac{aJ_1 \left[ 2a\sqrt{(U-\pi b)^2 + V^2} \right]}{\sqrt{(U-\pi b)^2 + V^2}} \right. \\
& \left. \cdot \sum_1^n \exp \left[ -i[p_n(U\cos\theta_n - \pi b\cos\theta_n + V\sin\theta_n)/a] \right] \right\} \\
& + \left\{ \frac{aJ_1 \left[ 2a\sqrt{(U+\pi b)^2 + V^2} \right]}{\sqrt{(U+\pi b)^2 + V^2}} \right. \\
& \left. \cdot \sum_1^n \exp \left[ -i[p_n(U\cos\theta_n + \pi b\cos\theta_n + V\sin\theta_n)/a] \right] \right\} \\
= & u_{1-b} + u_{1+b} \quad (2.8)
\end{aligned}$$

where  $a$  = radius of each subaperture,  $p_n$  = separation of subapertures from the origin of the multi-aperture system in terms of  $a$ ,  $b$  = point separation,  $\theta_n$  = angle from  $x$  axis which described the location of the center of each subaperture,  $n$  = number of subapertures in the multi-aperture system, and  $u_{1-b}$  was the field at the image plane due to the point source at  $-b$  on the  $x_0$  (object plane) axis and  $u_{1+b}$  was the field at the image plane due to the point source located at  $+b$  on the  $x_0$  axis.

The irradiance at the image plane,  $I_1(U,V)$ , was expressed as (4:109)

$$\begin{aligned}
I_1(U,V) &= |u_{1-b} + u_{1+b}|^2 \\
&= |u_1(U,V)|^2 \quad (2.9)
\end{aligned}$$

where  $u_1(U,V)$  was the complex amplitude computed in equation (2.8). Equations (2.8) and (2.9) were used to calculate the coherent far-field diffraction irradiance patterns at the image

plane for the three multi-aperture systems in this theoretical analysis.

### Incoherent Analysis

When a multi-aperture system was configured as in Figure 2.6 and illuminated by two closely-spaced incoherent point sources, the irradiance at the image plane,  $I_1(U,V)$ , was expressed as (4:110)

$$\begin{aligned} I_1(U,V) &= |u_{1-b}|^2 + |u_{1+b}|^2 \\ &= I_{1-b} + I_{1+b} \end{aligned} \quad (2.10)$$

where  $I_{1-b}$  was the irradiance in the image plane due to the point source located at  $-b$  on the  $x_0$  axis, and  $I_{1+b}$  was the irradiance in the image plane due to the point source located at  $+b$  on the  $x_0$  axis.  $u_{1-b}$  and  $u_{1+b}$  were found using equation (2.8) which described the complex field amplitude at the image plane due to two point sources at  $-b$  and  $+b$  on the  $x_0$  axis. Equations (2.8), (2.9), and (2.10) formed the basis for the calculations of the far field diffraction irradiance patterns which, in turn, were used to establish a two-point resolution criterion.

### III. Results

The following is a discussion and analysis of the results of the calculated diffraction patterns. Using these results, a two-point resolution criterion was established and its use demonstrated for three multi-aperture optical systems.

#### COMPUTER PROGRAM

The Fortran computer program that was used to compute the diffraction patterns is listed in Appendix A. The first portion of the program computes the diffraction patterns of the envelope function from each point source using the IMSL (9) subroutine MMBSJ1 to calculate the required Bessel functions. This was followed by the calculation of the field amplitudes caused by the two closely-spaced point source fields which propagated through the multi-aperture systems. The final portion of the program detected and stored the secondary side lobe maxima.

#### DATA COLLECTION AND ANALYSIS

Diffraction patterns from three multi-aperture optical

systems, at varying aperture and point separations, were calculated and analyzed. Figure 3.1 depicts the aperture systems. These three aperture systems, which were comprised of a varying number of apertures and aperture-origin separations, were chosen to demonstrate that the new two-point resolution criterion proposed in this paper would apply to any multi-aperture system. The aperture-origin separations of the six and four aperture systems varied from  $2.00a$  to  $5.00a$  ( $a$  = aperture radius) while the three aperture system varied from  $1.1547a$  to  $5.00a$ . Due to the geometry of three aperture system,  $1.1547a$  was the minimum aperture-origin separation necessary for the subapertures to just contact each other.

The initial portion of the theoretical analysis consisted of examining diffraction patterns from each multi-aperture configuration. The aperture-origin separations varied from where the apertures were just in contact with each other until the aperture-origin separation was a distance of  $5.00a$ , where  $a$  = aperture radius. At each aperture-origin separation, the separation of the point sources varied from a normalized distances of  $0.00$  to  $1.50$ . At each point separation, the diffraction pattern was analyzed to determine the irradiance of the central lobes and the maximum irradiance of the secondary side lobes. Figures 3.2a through c illustrate these diffraction patterns for several aperture systems illuminated incoherently by two point sources. In all cases, the maximum value of irradiance of each diffraction pattern was normalized to  $1.00$ . Figure 3.2a is the diffraction pattern for a single

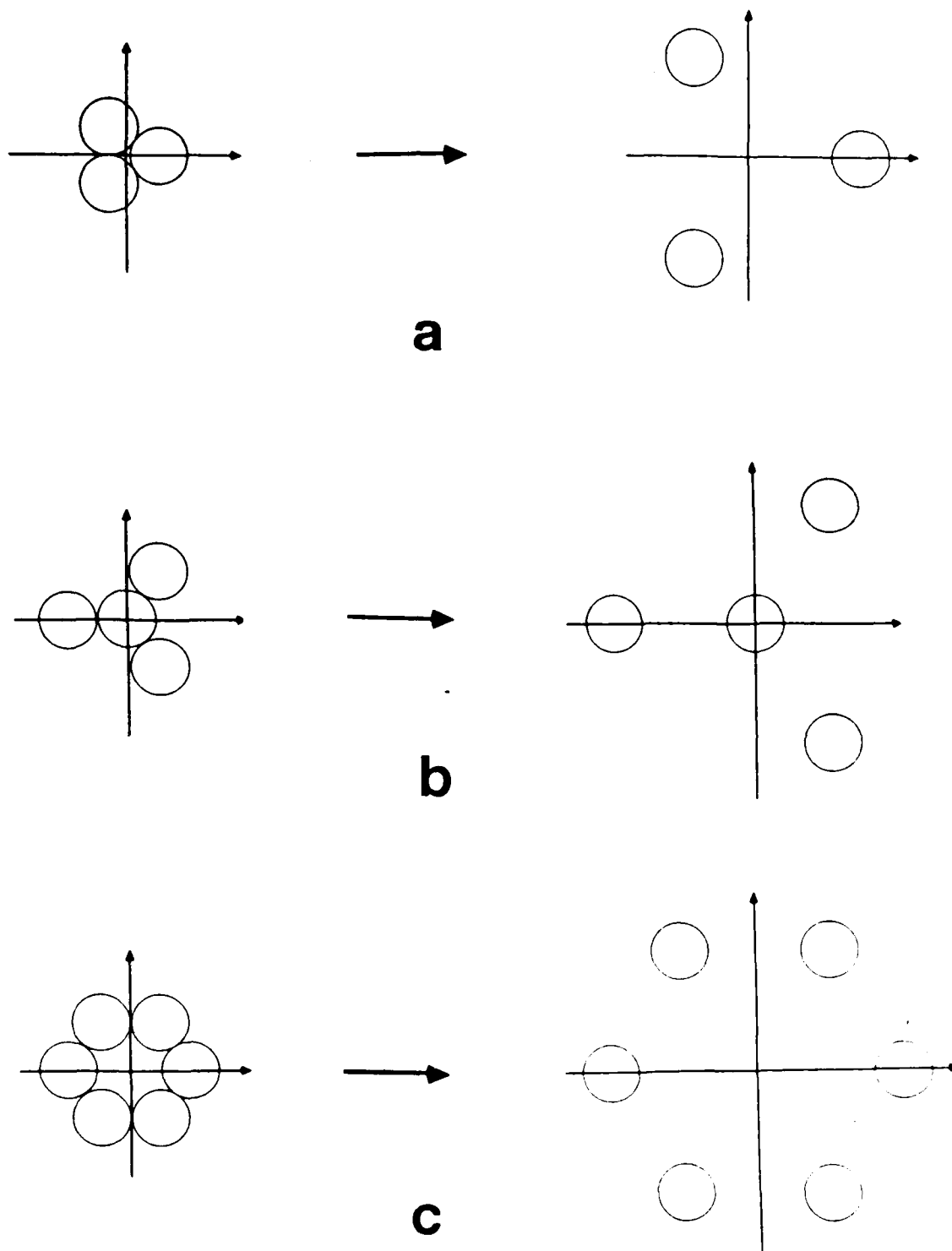


Fig. 3.1. Depiction of the a) Three, b) Four, and c) Six Aperture Systems.



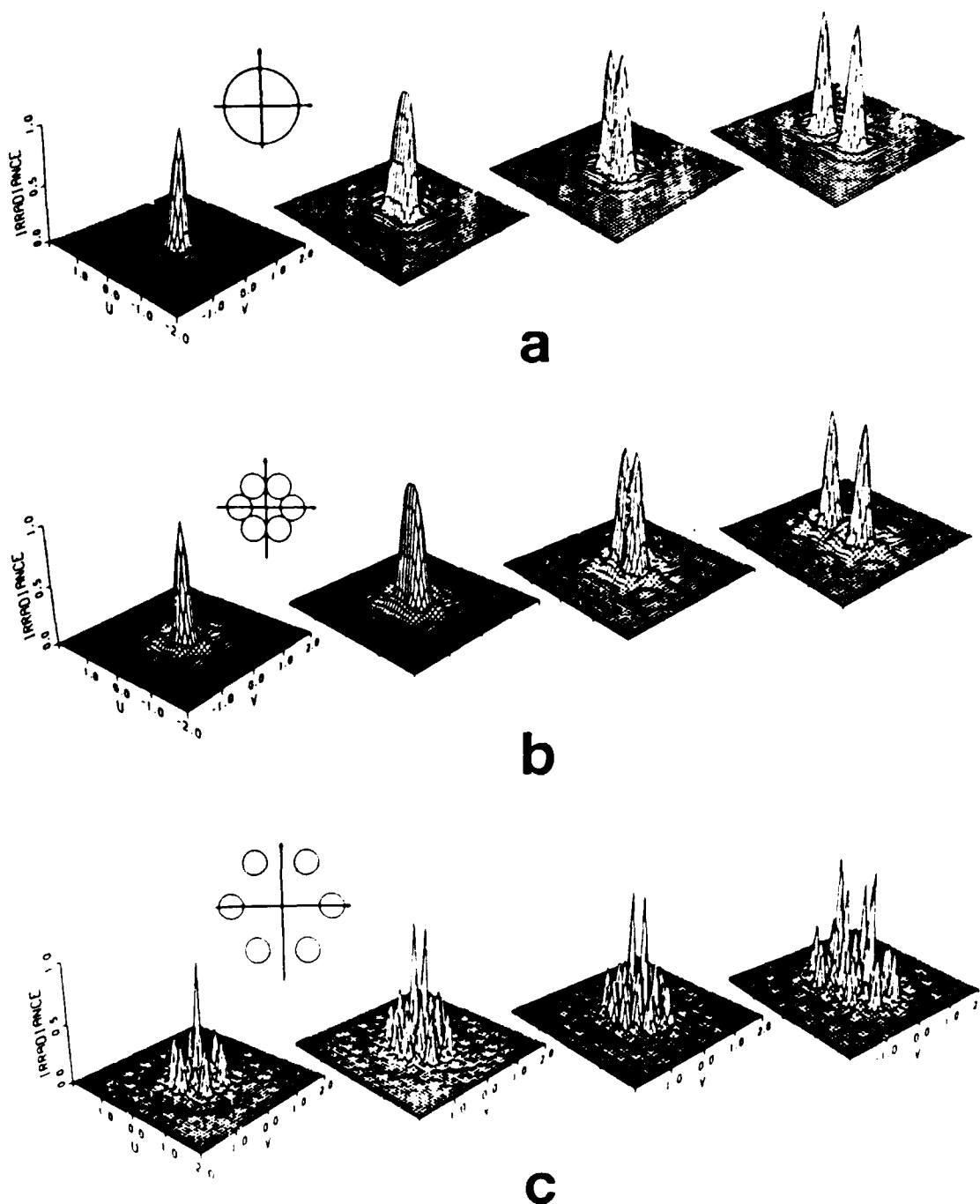


Fig. 3.2. Irradiance Patterns for Incoherent Two-Point Illumination of a) Single Large Aperture, Six Aperture Systems with Aperture-Origin Separations of b)  $2.00a$  and c)  $4.00a$ . Each Irradiance Pattern in each Series Corresponds to Point Separations of  $0.00$ ,  $0.16$  - Single Aperture Sparrow Limit,  $0.19$  - Single Aperture Rayleigh Limit, and  $0.50$ , respectively.

large aperture that has an equivalent diameter of a six aperture system with an aperture-origin separation of  $2.00a$ . Figures 3.2b and c are diffraction patterns for a six aperture system with aperture-origin separations of  $2.00a$  and  $4.00a$ , respectively. The normalized point separations for each series of patterns are 0.0, 0.16 (which is the Sparrow limit for the single aperture system), 0.19 (which is the Rayleigh limit for the single aperture system), and 0.5, respectively. The series of patterns from the single aperture system exhibited the summation of two Airy patterns. However, the patterns generated by the six aperture systems were considerably different. The diffraction patterns for a single point source illumination for these cases exhibited a central lobe with a normalized irradiance of 1.0 surrounded by six side lobes. As the point sources varied in separation, there was a variation in the individual side lobe irradiances due to the addition of the irradiances from each point source. For the case of the six aperture system with an aperture-origin separation of  $4.00a$  illuminated by the two point sources separated by 0.5 distance, the side lobe irradiances approached the irradiance value of the central lobes. Due to the existence of these side lobes, the two-point resolution criterion used for single aperture systems is not appropriate for the multi-aperture systems.

#### Irradiance versus Point Separation Plots

The data from the diffraction patterns for point

separations, which varied from 0.00 through 1.50 with 0.05 increments, and a single aperture-origin separation were collated on Irradiance versus Point Separation plots. Data were collected for aperture-origin separations of 2.00a through 5.00a at 0.25a increments for the four and six aperture systems. The values of aperture-origin separation for the three aperture system were 1.1547a through 5.00a. The values of irradiance on each plot were normalized such that the maximum irradiance value was equal to 1.0.

Figure 3.3 is an example of an Irradiance vs. Point Separation plot. This plot is a collection of data for a six aperture system with an aperture-origin separation of 2.00a and illuminated by two incoherent point sources. The solid curve represents the irradiance at the point which is the geometric midpoint between the imaged points of the two point sources (defined as the central value), or the squared modulus of the field amplitude at the same point if coherently illuminated. This central value remained equal to 1.0 from a point separation of 0.00 to the point separation where the Sparrow like criterion was met. Refer to Figure 3.2b for a depiction of the Sparrow criterion. Once the point separation was greater than the Sparrow like criterion, the central value began to decrease. That is, a dip in the irradiance between the now-resolved point images would deepen as the point separation was increased. Once the central value curve reached a minimum, it was terminated.

The dashed curve represents the maximum side lobe

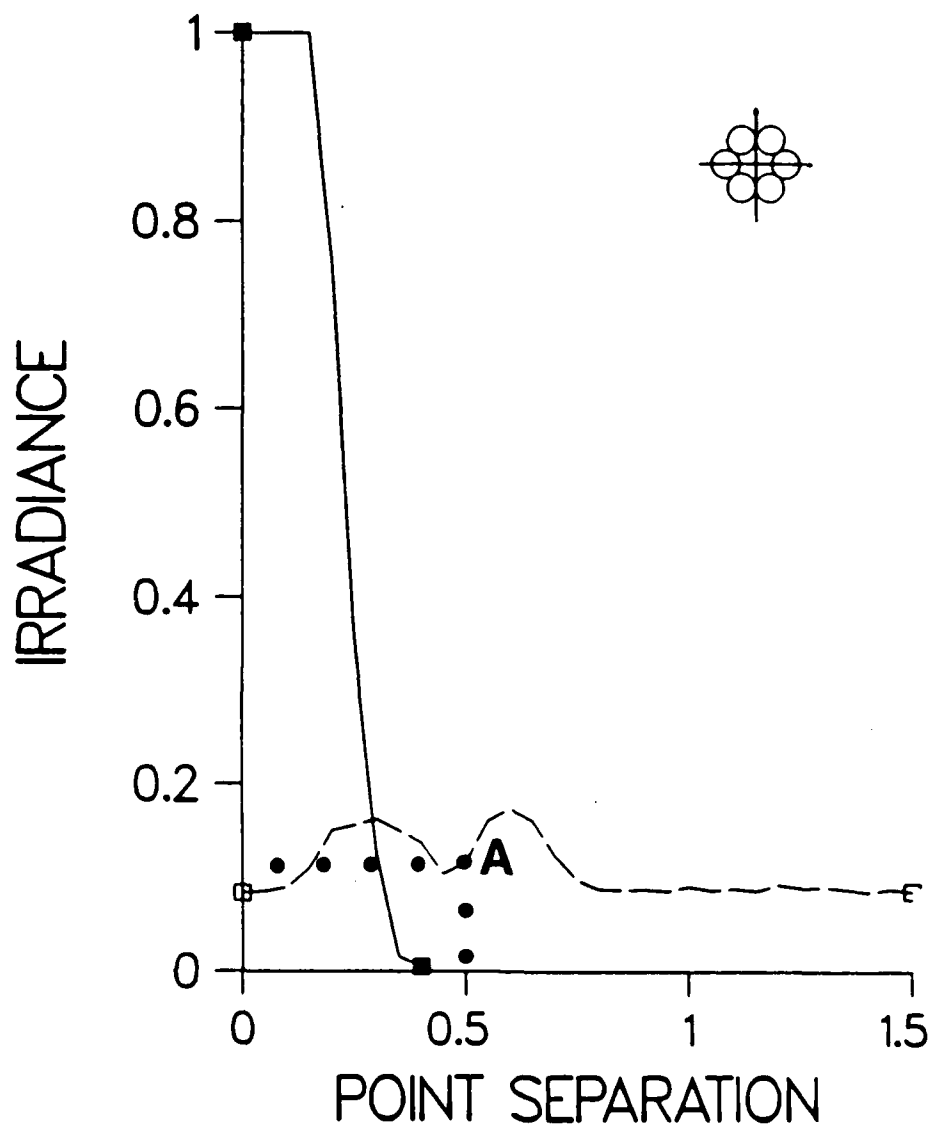


Fig. 3.3. Example Irradiance vs. Point Separation Plot for a Six Aperture System, Aperture-Origin Separation =  $2.00a$ , Illuminated Incoherently (\_\_\_\_ Central Value, - - - Side Lobe Maxima).

irradiance at a particular point separation. For example, point A in Figure 3.3 represents the data from Figure 3.2b which had a maximum side lobe irradiance of 0.11 at a point separation of 0.50. This value was placed on the Irradiance vs. Point Separation plot. This procedure was repeated for point separations of 0.00 through 1.50 in increments of 0.05.

When the aperture-origin separation became large enough, the side lobe maximum irradiance could be equal to or greater than the central lobe irradiance. This is evident in Figure 3.4 which displays the irradiance pattern for a six aperture system with an aperture-origin separation of  $4.00a$  and illuminated by two coherent point sources separated by 0.50 distance. The side lobe maximum irradiance is greater than the central lobe irradiance. This situation is represented on Figure 3.5, the Irradiance vs. Point Separation plot for this aperture system, as point A. Whenever the side lobe maxima exceeded the central lobe irradiances, the side lobe maxima was limited to a normalized value of 1.0.

#### TWO POINT RESOLUTION CRITERION - THRESHOLDS

The impulse response of a multi-aperture system is characterized by a narrow central lobe surrounded by side lobes. When adding the fields or irradiances from two point sources propagated through a multi-aperture system, the side lobe irradiances could equal or exceed the height of the central lobes. As a result, two-point resolution is not solely

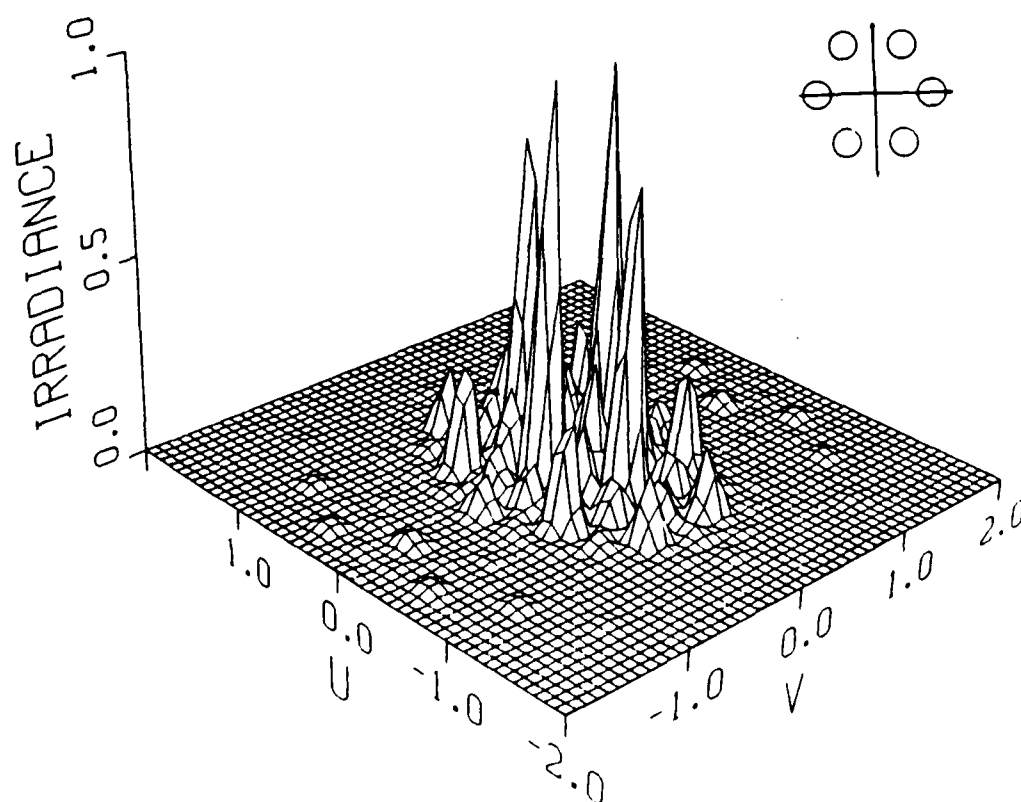


Fig. 3.4. Irradiance Pattern for a Six Aperture System Illuminated Coherently with Aperture-Origin Separation =  $4.00a$  and Point Separation =  $0.50$ .

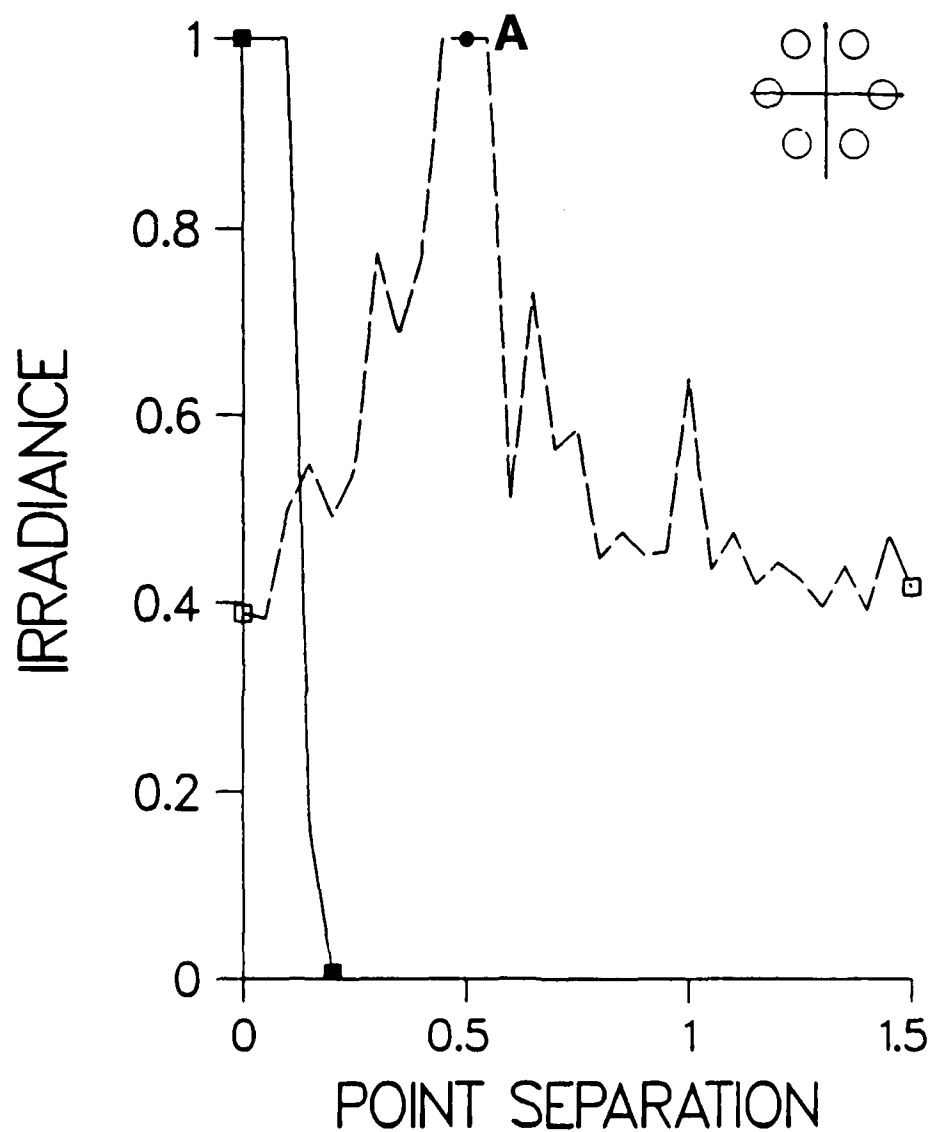


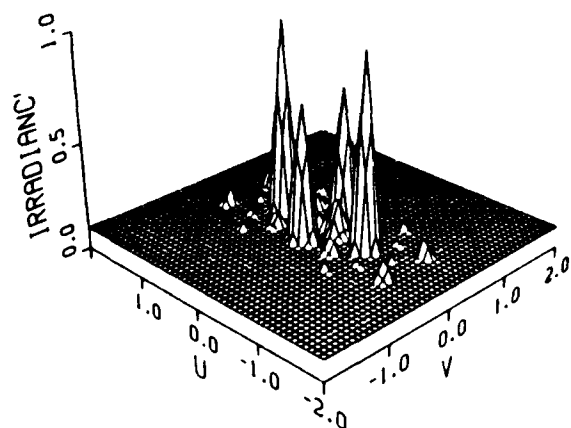
Fig. 3.5. Irradiance vs. Point Separation Plot for a Six Aperture System, Aperture-Origin Separation =  $4.00a$ , Illuminated Coherently (— Central Value, — — — Side Lobe Maxima).

a matter of central lobe width, but is also dependent upon the height of the side lobes. The single aperture two-point resolution criterion does not take into account the effect of these side lobes. Because of this problem, a new two-point resolution criterion was proposed.

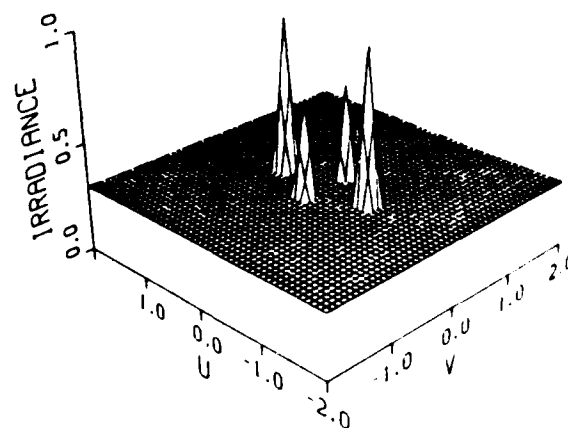
The new two-point resolution criterion proposed in this paper is based on the idea of thresholds. The threshold is defined as a fraction of the central lobe irradiance. Any lobe irradiance which is greater than the specified threshold is observed while any with irradiances less than the specified threshold are disregarded. The thresholds vary from 0.1 to 0.9 of the central lobe irradiance of the far-field diffraction patterns.

Figures 3.6a through d illustrate the threshold idea using diffraction patterns for a six aperture system with an aperture-origin separation of  $3.00a$  which was illuminated by two closely-spaced coherent point sources. The two point sources were separated by a normalized distance of 0.70. The grid, which is a plane parallel to the U-V plane, represents the threshold value. Figure 3.6a represents the threshold at a value of 0.1. At this threshold, there existed a substantial number of side lobes. However, as the threshold value increased, the number of secondary lobes that were greater than the threshold value decreased. When the 0.9 threshold was reached, the only two diffraction lobes which remained were the central lobes which represented the detected location of the two point images which were clearly resolved.

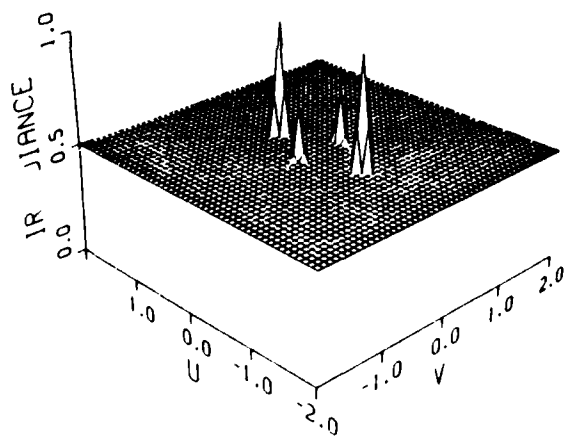
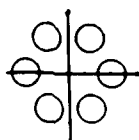




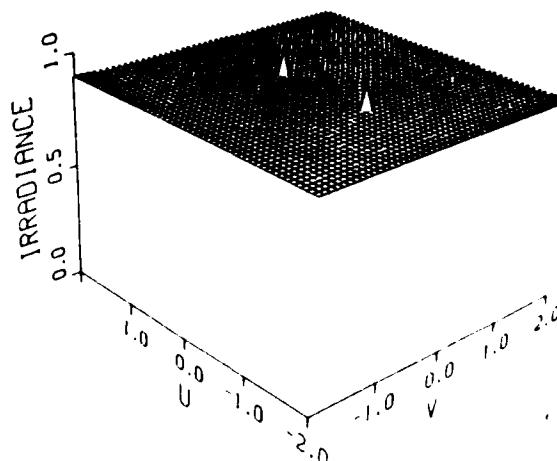
**a**



**b**



**c**



**d**

Fig. 3.6. Diffraction Patterns Limited by Threshold Values of a) 0.1, b) 0.3, c) 0.5, and d) 0.9.

Figures 3.7a through d depict the diffraction patterns for a six aperture system with an aperture-origin separation of  $3.00a$  illuminated by two coherent point sources and limited by a threshold value of 0.5. The two point sources varied in separation such that the imaged points moved from being clearly resolved to unresolved. Figure 3.7a represents the point source separation of 1.50. At this point separation, the imaged point sources are clearly resolved. However, at point separations of 0.70 and 0.65, depicted in Figures 3.6b and c, respectively, the side lobes have irradiances greater than the 0.5 threshold value, and, as a result, the two imaged point sources are no longer resolved. Figure 3.6d depicts the situation where the point sources have moved to a point separation of 0.30. According to this diffraction pattern with the imposed threshold value, the imaged points are resolved. The phenomenon of the two point sources passing in and out of resolution is evident in the Irradiance vs. Point Separation plots presented in this paper. However, the new two-point resolution criterion that is being proposed is conservative. The point separation where two point sources were considered resolved was determined by moving the two point sources from an infinite point separation towards a 0.00 point separation. Once the two point sources became unresolved, they were considered unresolved at all lesser point separations.

To establish the threshold criterion, the Irradiance vs. Point Separation plots were analyzed. As an example, consider the same six aperture system with an aperture-origin separation

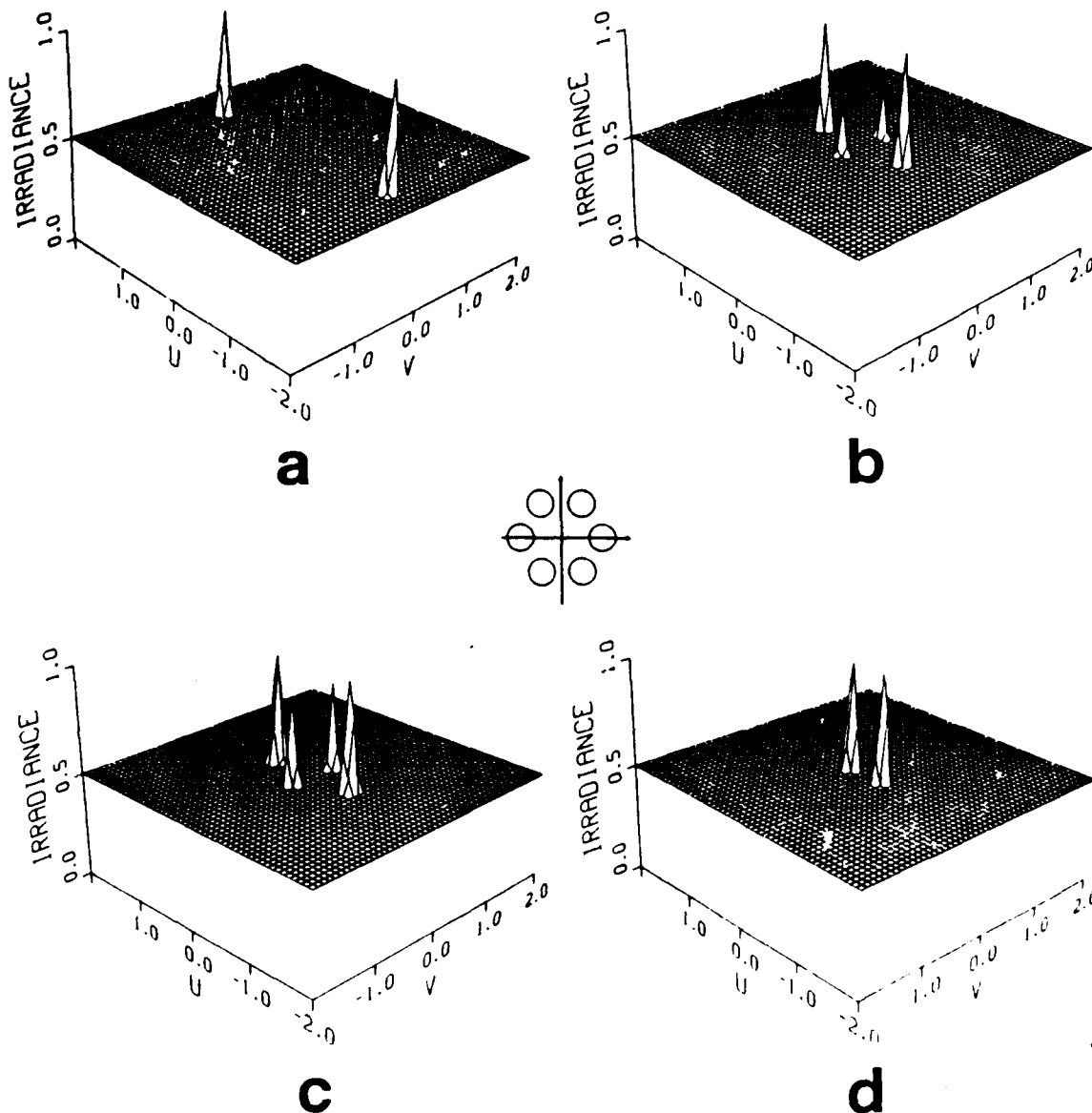
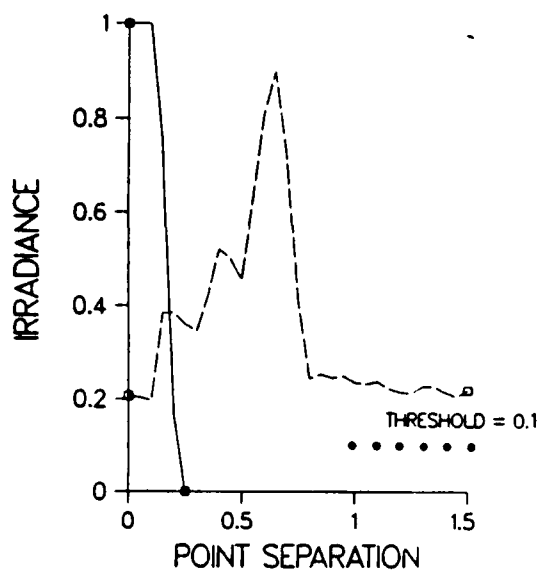


Fig. 3.7. Diffraction Patterns of a Six Aperture System with an Aperture-Origin Separation of  $3.00a$  Illuminated by Two Coherent Point Sources and Limited by a Threshold Value of 0.5 at Point Separations of a) 1.50, b) 0.70, c) 0.65, and d) 0.30.

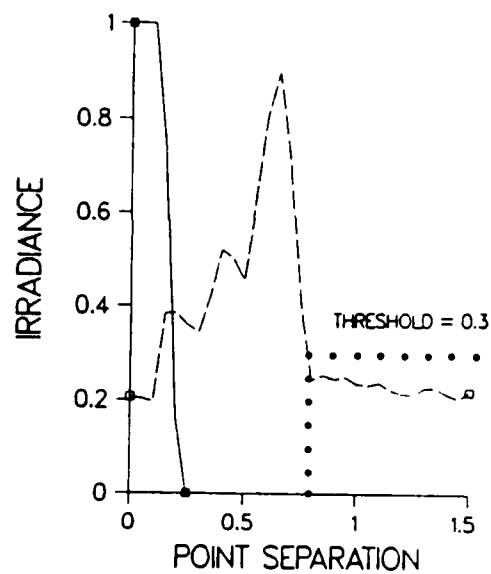
of 3.00a (Figure 3.1c) which was illuminated by two closely-spaced coherent point sources. The Irradiance vs. Point Separation plot which represents this configuration is presented in Figure 3.8a. A dotted line has been added to this figure which represents 0.1 the value of the central lobe irradiance. This is the 0.1 threshold value. Since all of the side lobe maxima are greater than the 0.1 threshold value at all point separations, the two central lobes cannot be resolved from the surrounding side lobes. As a result, it is not possible to resolve the two point sources at any point separation.

Figure 3.8b represents the 0.3 threshold value. At 0.3 of the central lobe irradiance, this multi-aperture system with a 3.00a aperture-origin separation was able to resolve the two central lobes of the diffraction pattern at a point separation of approximately 0.80 or greater. Figure 3.8c represents the 0.7 threshold value. This multi-aperture system was able to resolve the two central lobes as long as the two point separation was at least 0.70.

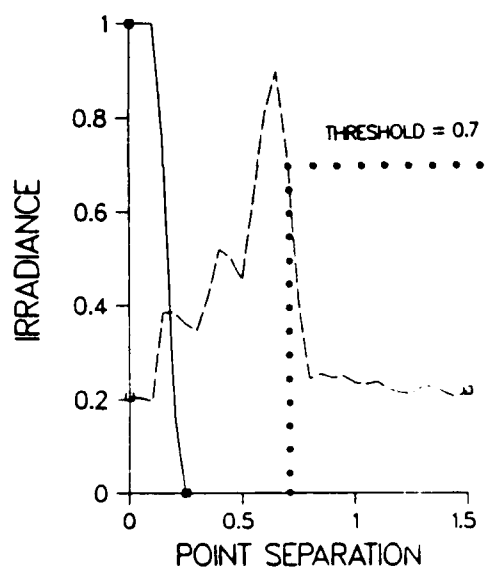
Figure 3.8d depicts the case when the threshold value was greater than the maximum side lobe irradiance at any point separation. When this situation occurred, the two-point resolution criterion for multi-aperture systems reduced to the Sparrow like criterion.



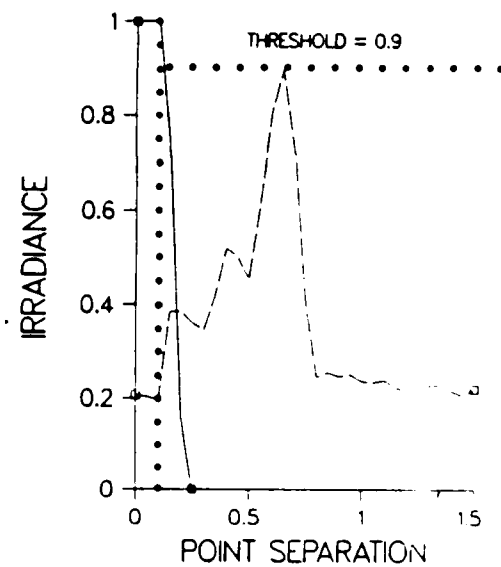
**a**



**b**



**c**



**d**

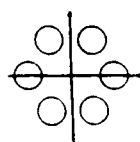


Fig. 3.8. Illustration of Threshold Criterion for Coherently Illuminated, Six Aperture System, Aperture-Origin Separation =  $3.00a$ , and Threshold Values of a) 0.1, b) 0.3, c) 0.7, and d) 0.9 (— Central Value, --- Side Lobe Maxima, ..... Threshold Value).

## Threshold Plots

The threshold plots introduced in this section are collations of data from the Irradiance vs. Point Separation plots using the threshold analysis. There is one threshold plot for each type of multi-aperture system at various aperture-origin separations. The point separation where two-point resolution is achieved at a specific threshold value is plotted for each aperture-origin separation.

Figure 3.9 is a demonstration of how a threshold plot was compiled for a threshold value of 0.3. For this example, an six aperture system was illuminated by two coherent point sources. The aperture-origin separation varied from  $2.00a$  through  $5.00a$  in increments of  $0.25a$ . For each aperture-origin separation, the point separation where two-point resolution exists was plotted for a threshold value of 0.3. As the aperture-origin separation increased, the point separation where one was able to resolve the two points generally increased for this particular threshold value. For instance, at an aperture-origin separation of  $2.00a$ , the two points were resolved at a point separation of 0.65. At an aperture-origin separation of  $3.00a$ , the two points were resolved at a point separation of 0.75. As the aperture-origin separation increased above  $3.00a$ , the resolution limit also increased. As stated earlier in this paper, this new two-point resolution criterion is conservative and is displayed as such in all threshold plots.

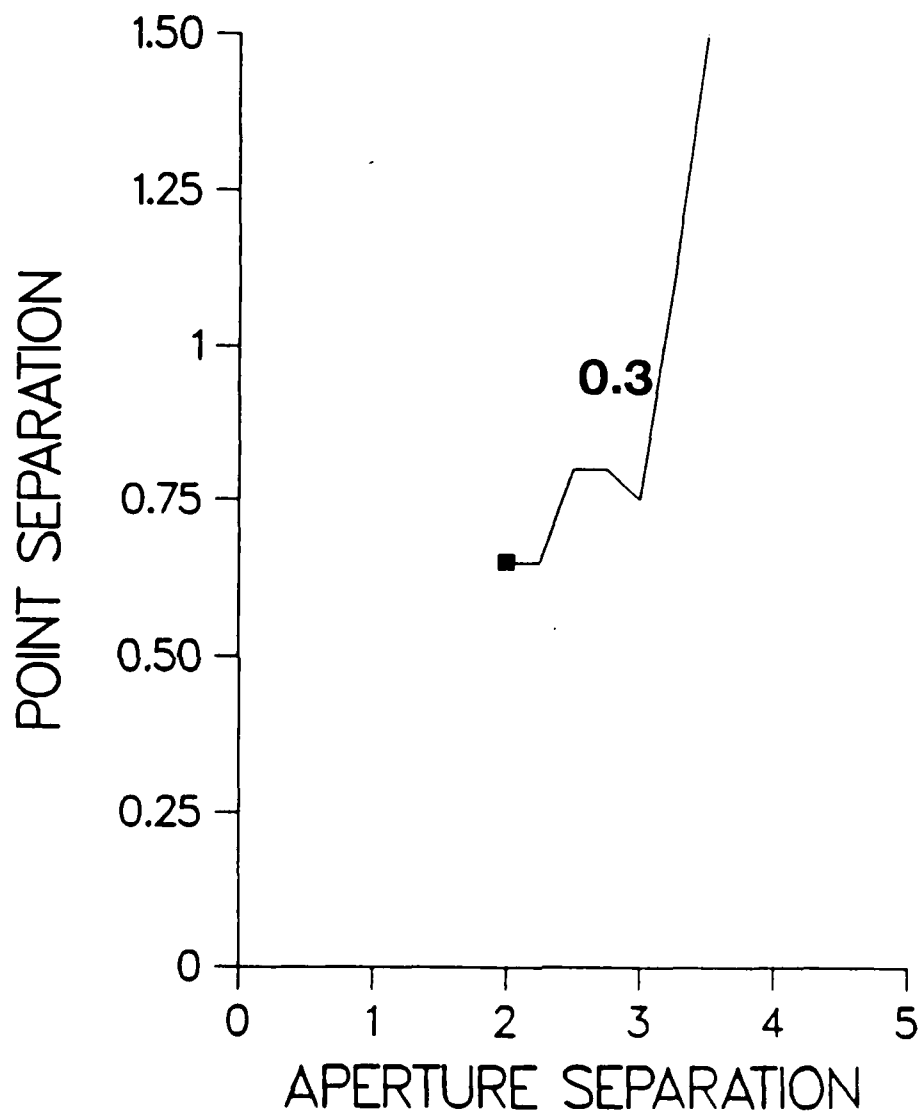


Fig. 3.9. Example Threshold Plot for Coherently Illuminated Six Aperture System at Threshold of 0.3.

The two-point resolution points for various aperture configurations were found for the threshold values of 0.1, 0.3, 0.5, 0.7, and 0.9 and placed on one threshold plot. Figure 3.10 is an example of a complete threshold plot for a six aperture system illuminated by two coherent point sources. The threshold curves were computed using the same method as that for the 0.3 threshold curve depicted in Figure 3.9. Generally, as the threshold value increased for a particular aperture-origin separation, the point separation necessary to resolve two point sources decreased. This is the form of the data that will be used for the analyses presented in this paper.

## RESULTS - THRESHOLD TWO-POINT RESOLUTION CRITERION

Three specific multi-aperture systems were analyzed to demonstrate the proposed two-point resolution criterion using the threshold idea. Each system was examined with both coherent and incoherent illumination.

### Three Aperture System

For the three aperture system, the aperture-origin separation was varied from  $1.1547a$  through  $5.00a$ . Figures 3.11a through d are Irradiance vs. Point Separation plots for aperture-origin separations of  $1.1547a$ ,  $2.00a$ ,  $3.00a$ , and  $4.00a$  which were coherently illuminated. The side lobe maxima



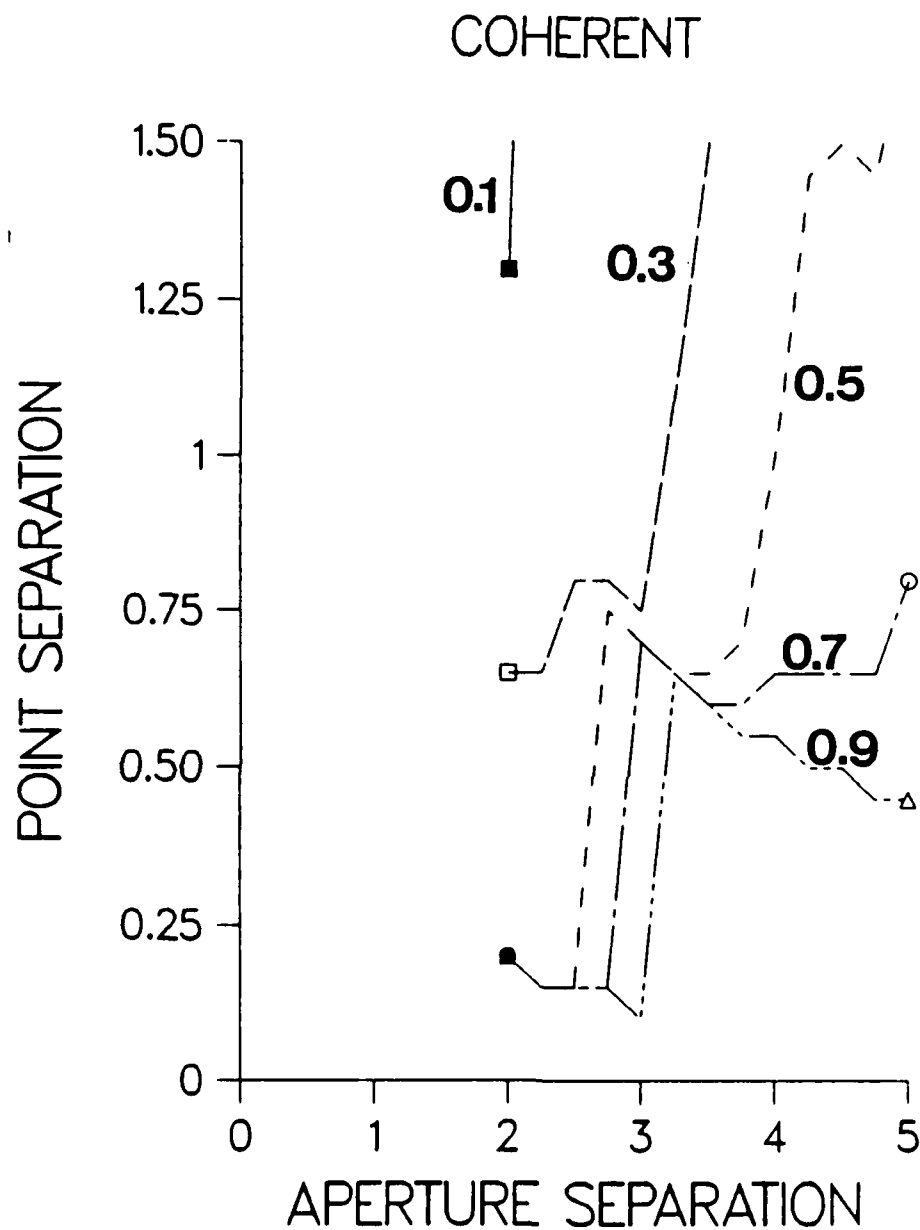
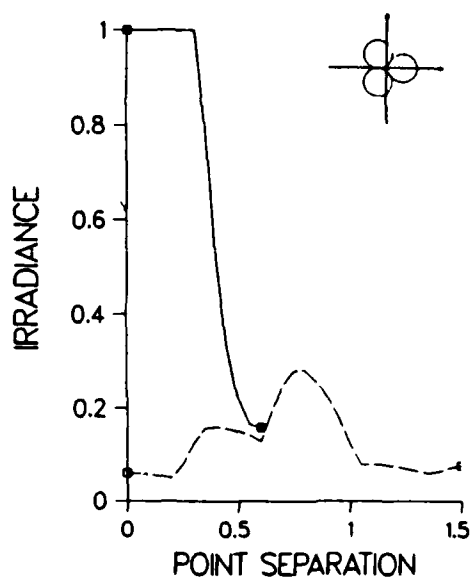
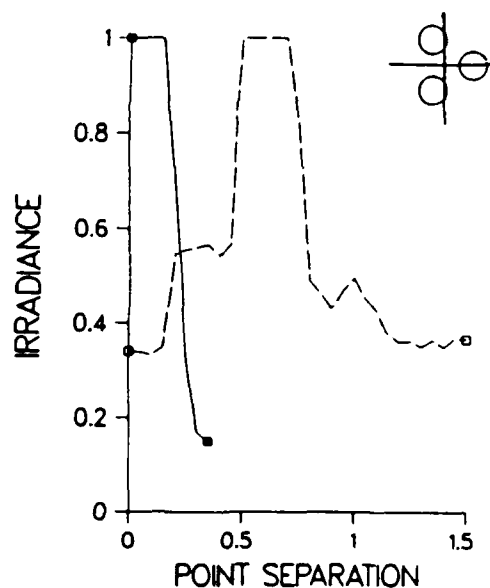


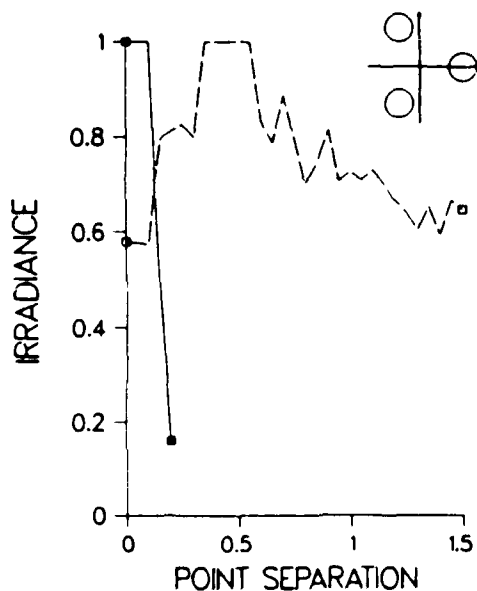
Fig. 3.10. Example Threshold Plot for Coherently Illuminated Six Aperture System (Threshold Values:        0.1,        0.3,        0.5,        0.7,        0.9).



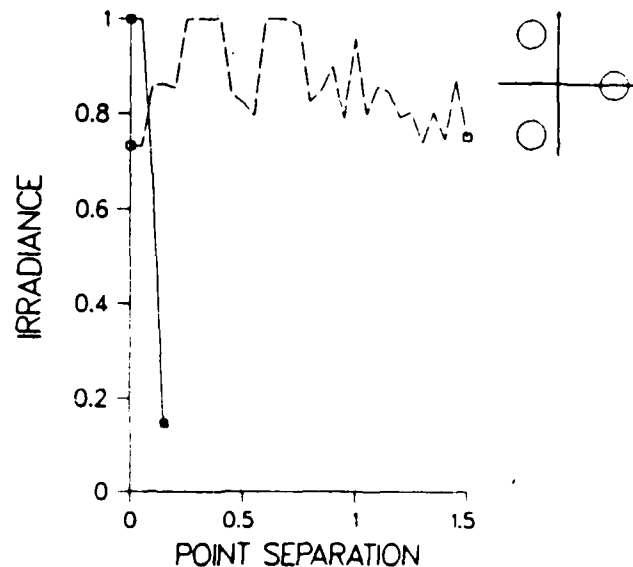
**a**



**b**



**c**

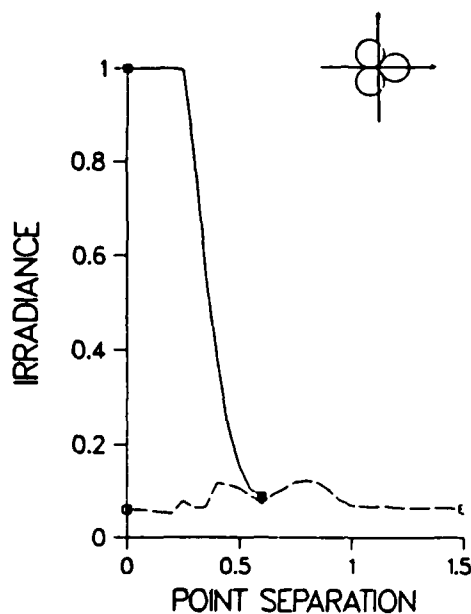


**d**

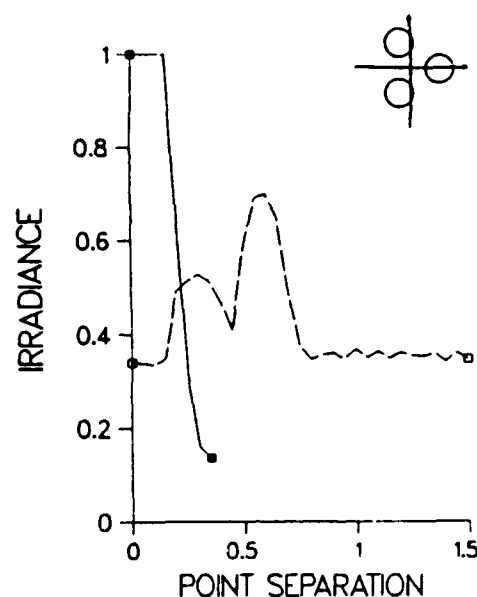
Fig. 3.11. Irradiance vs. Point Separation Plots for Coherently Illuminated Three Aperture System with Aperture-Origin Separations of a) 1.1547a, b) 2.00a, c) 3.00a, and d) 4.00a (— Central Value, --- Side Lobe Maxima).

increased rapidly as the aperture-origin separation increased. Figures 3.12a through d are the same plots for the aperture configuration except that the system was incoherently illuminated. Note the decrease in the side lobe maxima as compared to the coherently illuminated system. Note, also, the increase in resolution (Sparrow criterion of the central lobe) as the dilution increased. This was a result of the narrowing of the central lobes as the aperture-origin separation increased.

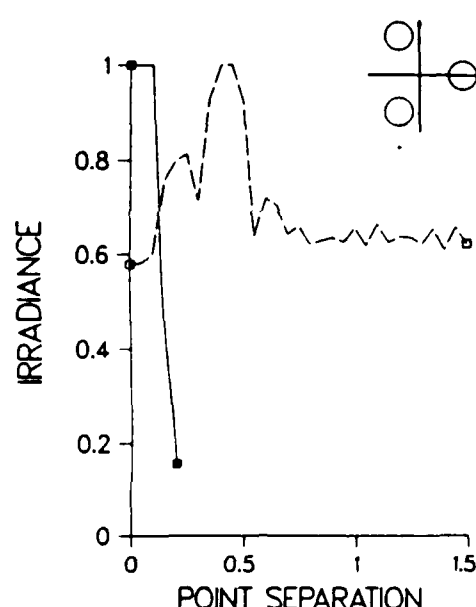
The threshold plots for the coherent and incoherent illumination of the three aperture system are presented in Figures 3.13a and b. These threshold plots allow a comparison of the two-point resolution performance of the coherently and incoherently illuminated three aperture system. In this case, Figures 3.13a and b indicate that for a given aperture-origin separation for coherent and incoherent illumination, the incoherently illuminated system had superior two-point resolution performance at all thresholds. For instance, at a threshold value of 0.9 and an aperture separation of  $2.00a$ , the incoherent system was able to resolve two point sources separated by a normalized distance of 0.15; whereas the coherent system required a normalized point separation of 0.70 to resolve the two points. This was a result of the relative decrease of the side lobe maxima when the three aperture system was illuminated incoherently rather than coherently. For incoherent illumination, the irradiances from the two point sources are summed. However, For coherent illumination, the



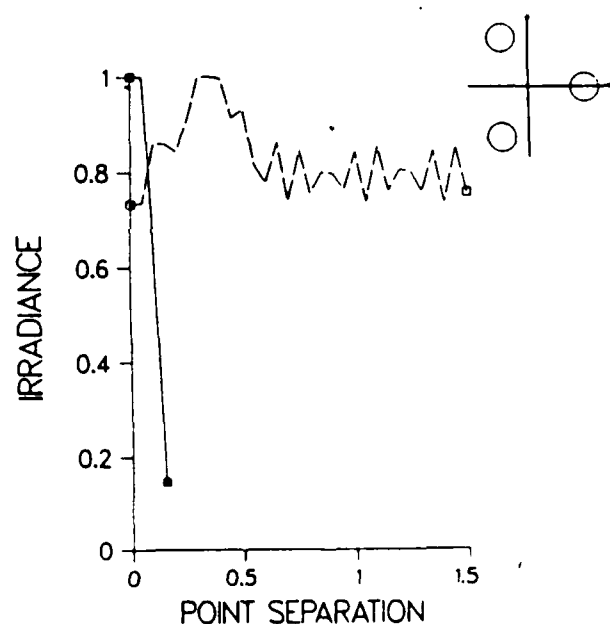
**a**



**b**



**c**



**d**

Fig. 3.12. Irradiance vs. Point Separation Plots for Incoherently Illuminated Three Aperture System with Aperture-Origin Separations of a) 1.1547a, b) 2.00a, c) 3.00a, and d) 4.00a (— Central Value, --- Side Lobe Maxima).

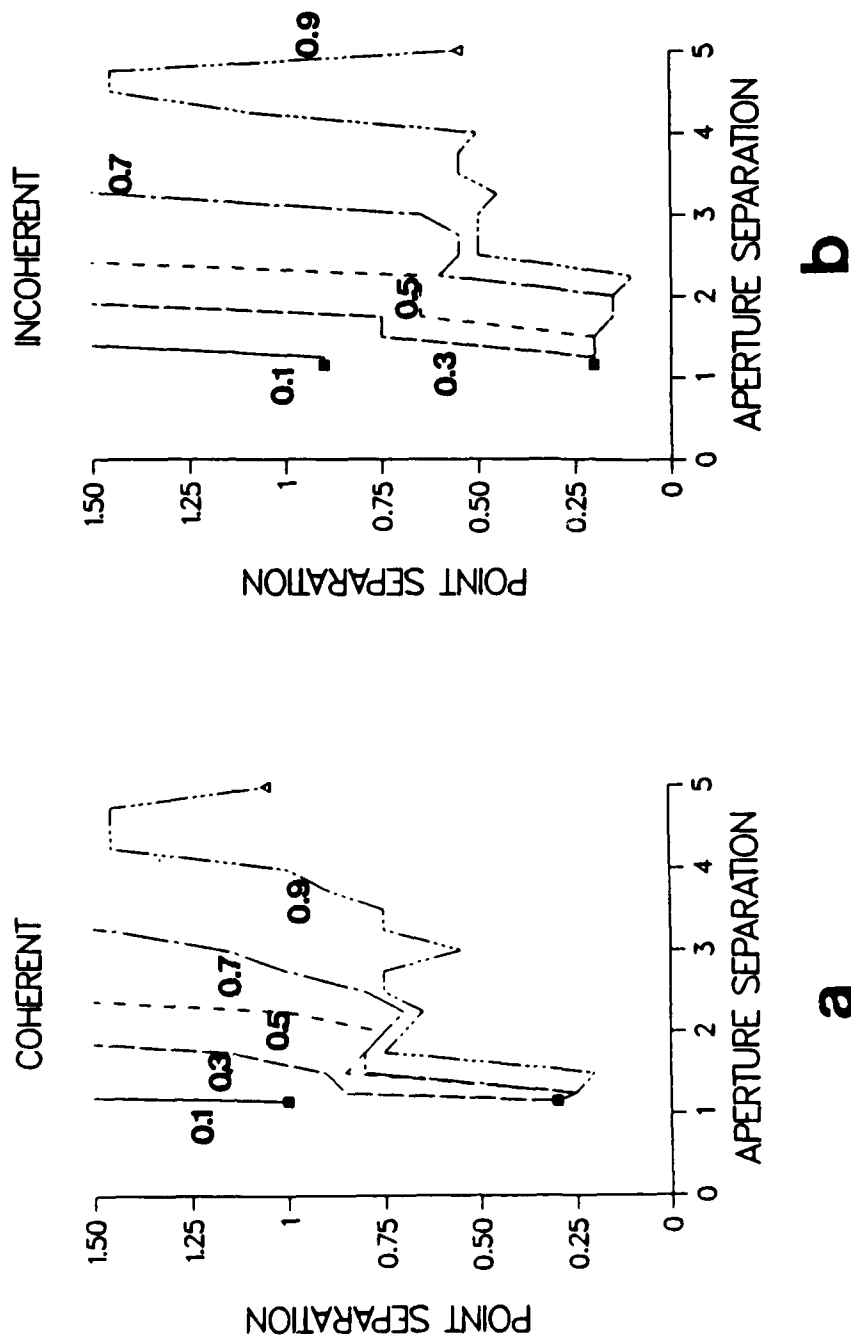


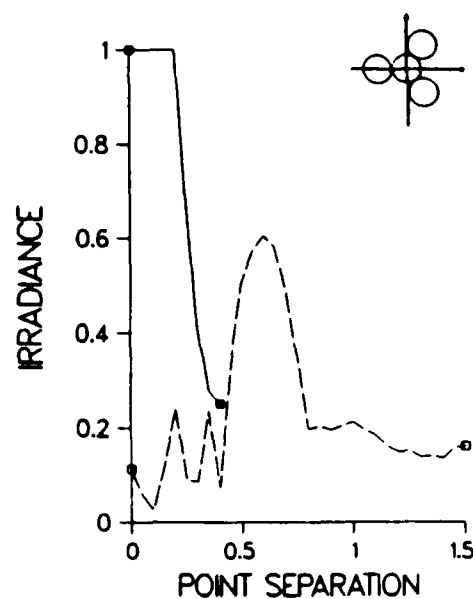
Fig. 3.13 Threshold Plots for Three Aperture System Illuminated a) Coherently and b) Incoherently (Threshold Values: 0.1, 0.3, 0.5, 0.7, 0.9).

field amplitudes from the point sources are added and then squared. This can cause greater side lobe irradiances when compared to the incoherent case.

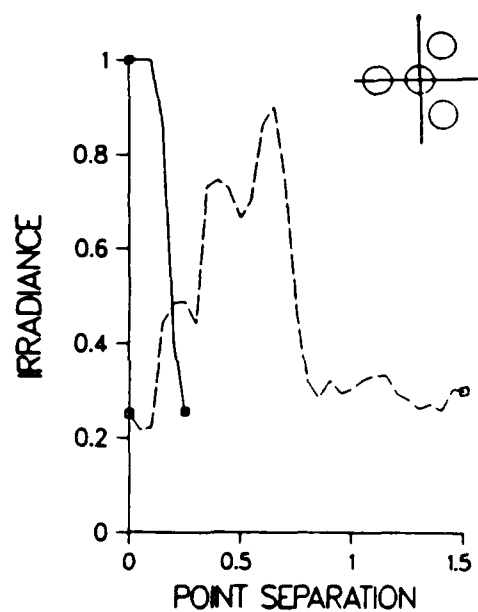
#### Four Aperture System

For the four aperture system, the aperture-origin separation was varied from  $2.00a$  through  $5.00a$  in increments of  $0.25a$ . Figures 3.14a through d are the Irradiance vs. Point Separation plots for the  $2.00a$ ,  $3.00a$ ,  $4.00a$ , and  $5.00a$  aperture-origin separations which were coherently illuminated. As described in the three aperture system, the increase of the side lobe irradiance was evident as the aperture-origin separation increased. Figures 3.15a through d are the Irradiance vs. Point Separation plots for the same aperture system illuminated incoherently. As was noted in the three aperture case, the side lobe maxima generally decreased as compared to the coherently illuminated system. In addition, there was increased resolution (Sparrow criterion) as the aperture-origin separation increased. This was also noted in the three aperture system.

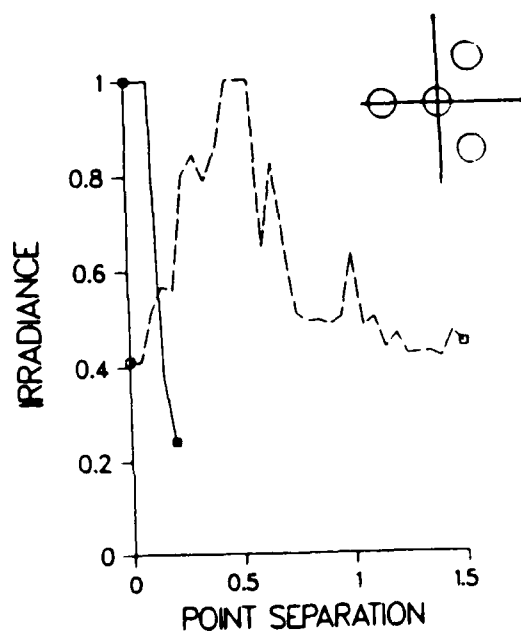
The threshold plots for the coherently and incoherently illuminated four aperture system are presented in Figures 3.16a and b, respectively. As demonstrated in the three aperture case, these plots allow a comparison of the ability of the four aperture system to resolve two point sources at various aperture-origin separations. Generally, the incoherent



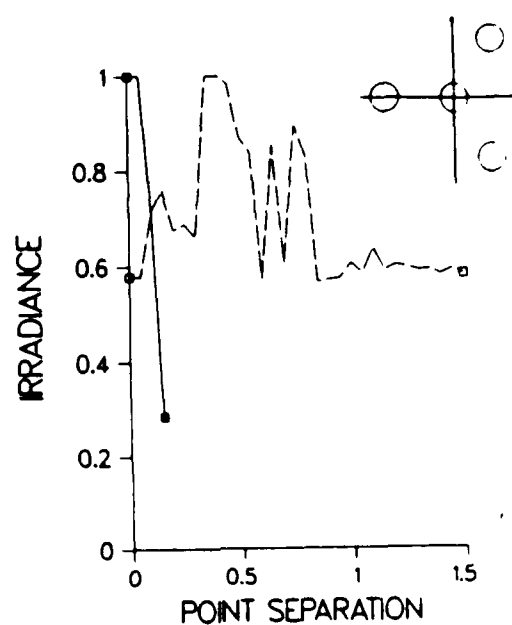
**a**



**b**

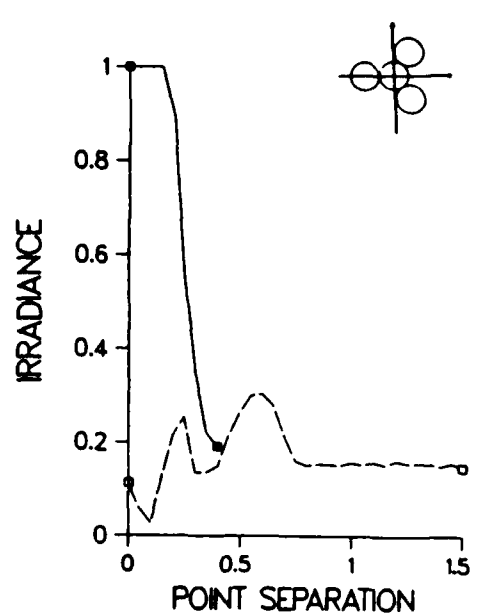


**c**

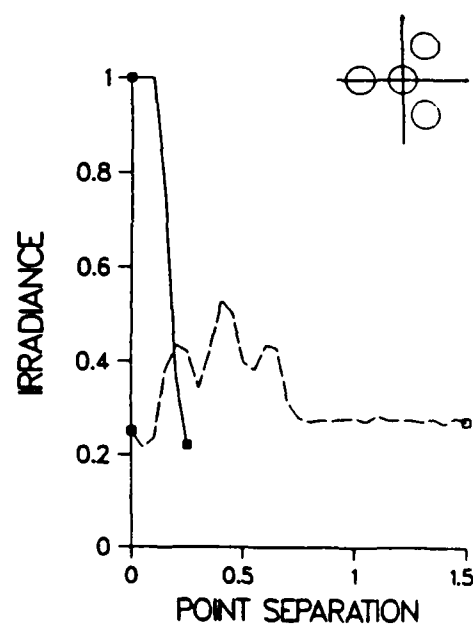


**d**

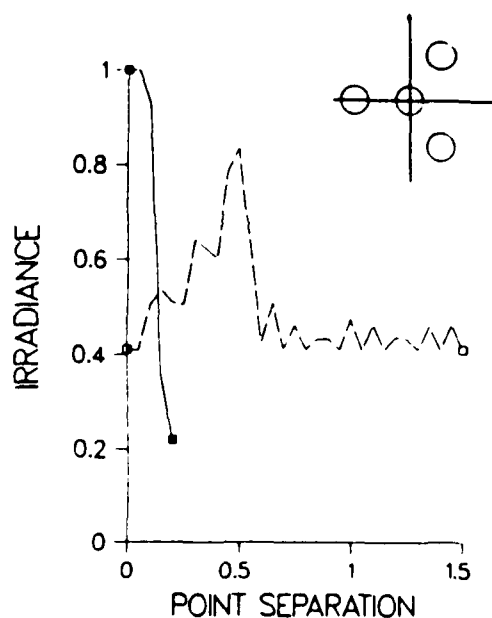
Fig. 3.14. Irradiance vs. Point Separation Plots for Coherently Illuminated Four Aperture System with Aperture-Origin Separations of a) 2.00a, b) 3.00a, c) 4.00a, and d) 5.00a (— Central Value, — — — Side Lobe Maxima).



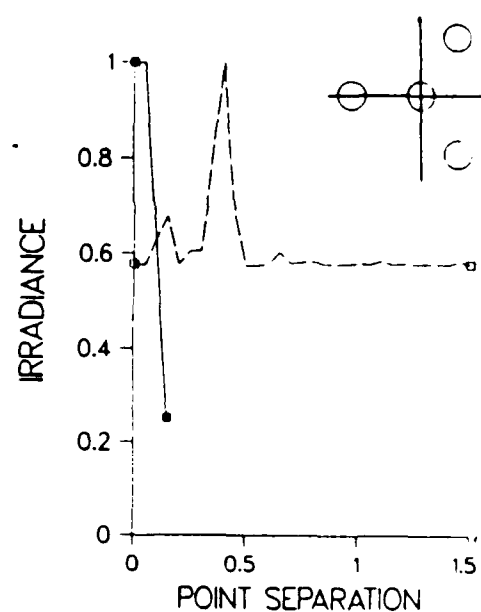
**a**



**b**



**c**



**d**

Fig. 3.15. Irradiance vs. Point Separation Plots for Incoherently Illuminated Four Aperture System with Aperture-Origin Separations of a)  $2.00a$ , b)  $3.00a$ , c)  $4.00a$ , and d)  $5.00a$  (— Central Value, — — — Side Lobe Maxima).



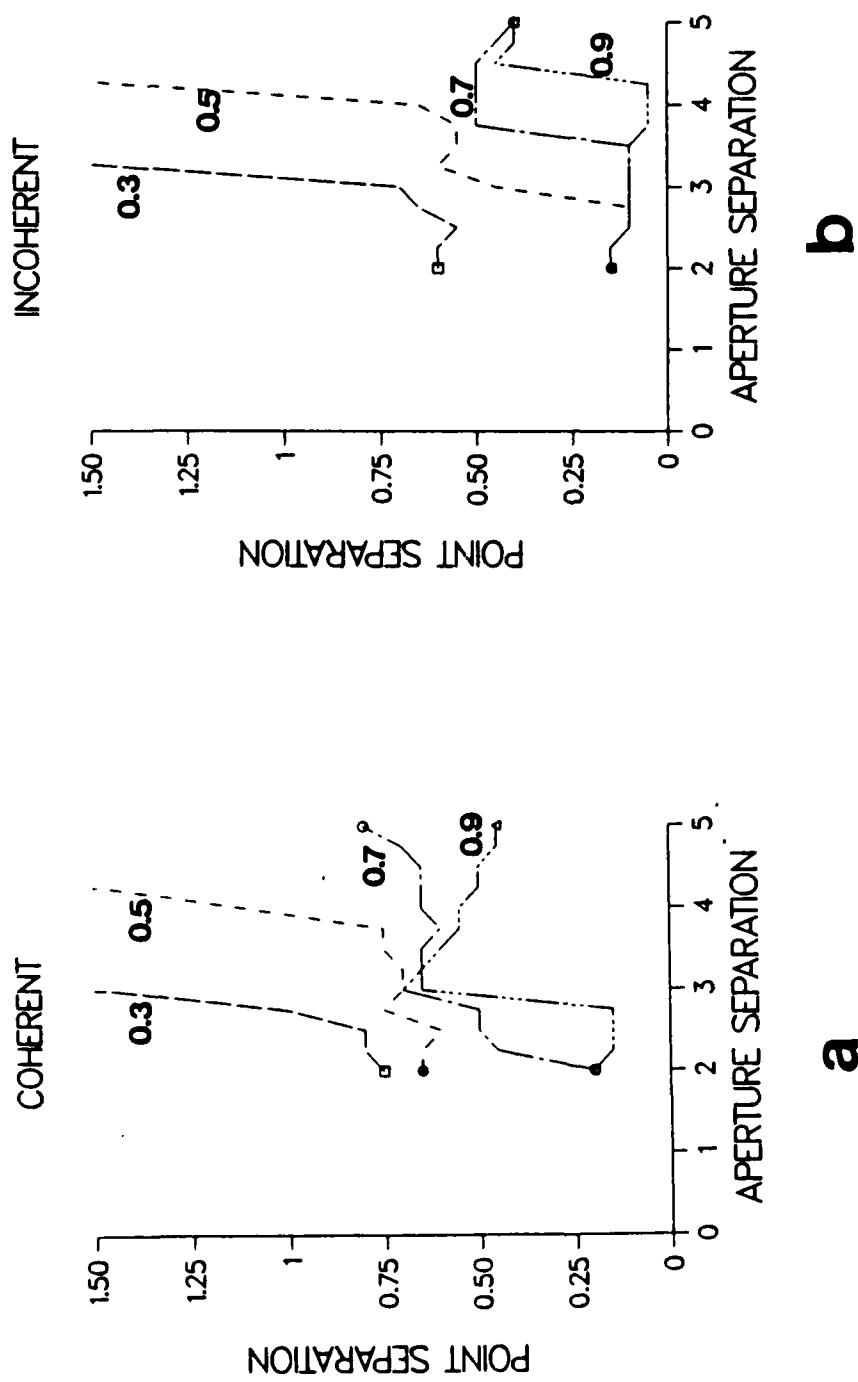


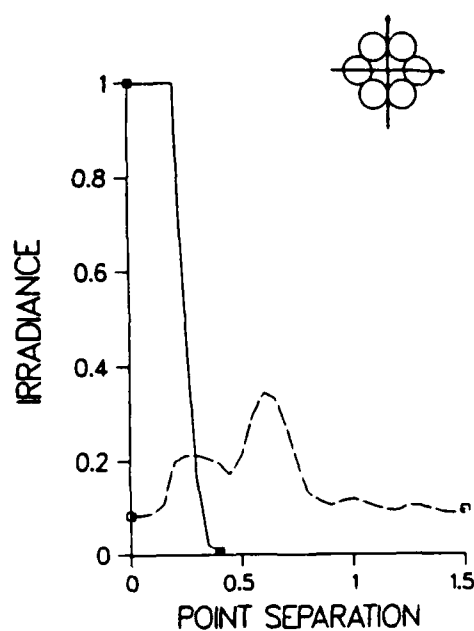
Fig. 3.16. Threshold Plots for Four Aperture System  
Illuminated a) Coherently and b) Incoherently  
(Threshold Values:  $\text{---}$  0.3,  $\text{---}$  0.5,  $\text{---}$  0.7,  $\text{---}$  0.9).

two-point resolution performance was superior to the coherent performance as a result of the relative decrease in the side lobe maxima. The superior performance of the incoherently illuminated system is most evident in the 0.7 and 0.9 threshold curves. At these thresholds, the incoherently illuminated system was able to resolve two point sources at point separations considerably less than in the coherently illuminated case.

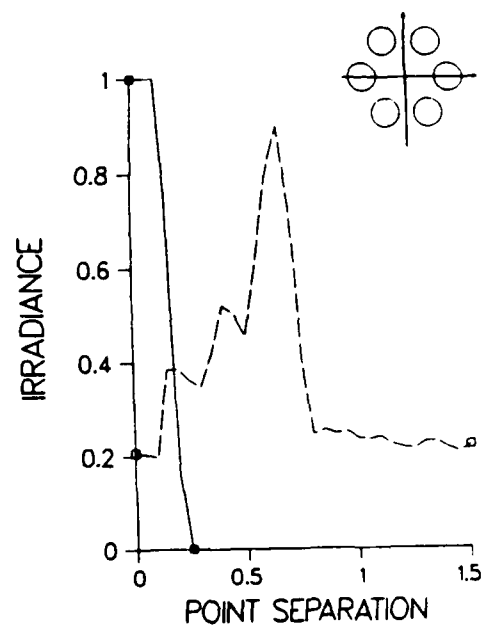
#### Six Aperture System

In the six aperture system, the aperture-origin separation varied from  $2.00a$  through  $5.00a$  in increments of  $0.25a$ . Figures 3.17a through d are the Irradiance vs. Point Separation plots for the  $2.00a$ ,  $3.00a$ ,  $4.00a$ , and  $5.00a$  aperture-origin separations which were coherently illuminated. Figures 3.18a through d are the same plots for incoherent illumination. As described in the three and four aperture cases, the side lobe maxima increased considerably as the aperture-origin separation increased. In addition, the side lobe maxima for the incoherently illuminated system were less than that for the coherently illuminated system. Also, the point separation necessary to satisfy the Sparrow criterion decreased as the aperture-origin separation increased.

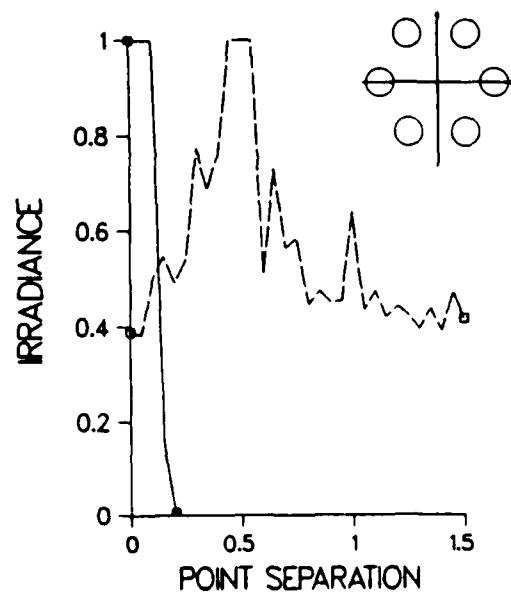
The threshold plots for the six aperture system illuminated coherently and incoherently are depicted in Figures 3.19a and b, respectively. As was illustrated in the three and



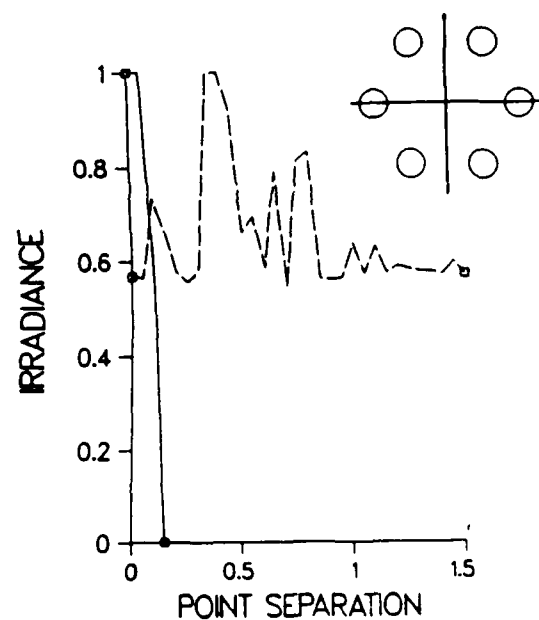
**a**



**b**

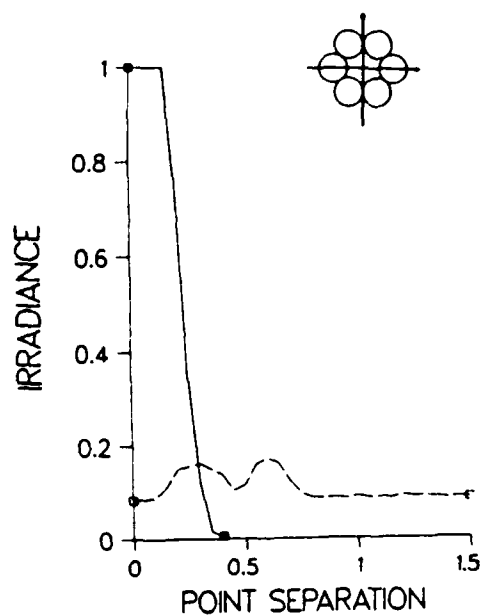


**c**

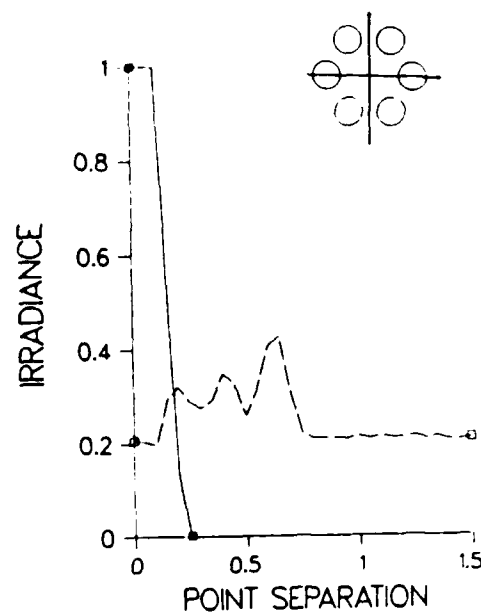


**d**

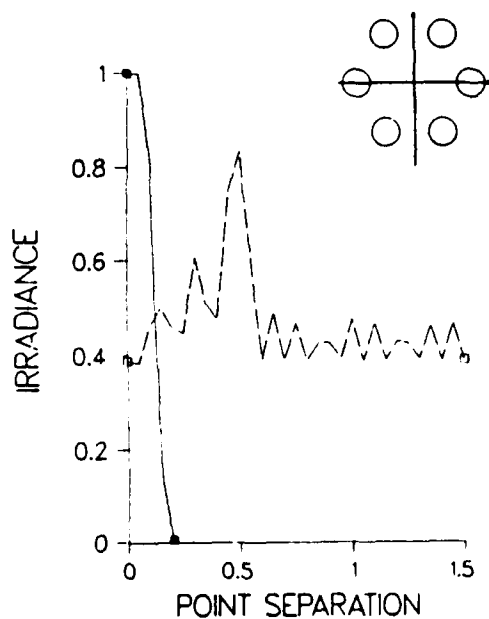
Fig. 3.17. Irradiance vs. Point Separation Plots for Coherently Illuminated Six Aperture System with Aperture-Origin Separations of a)  $2.00a$ , b)  $3.00a$ , c)  $4.00a$ , and d)  $5.00a$  (— Central Value, — — — Side Lobe Maxima).



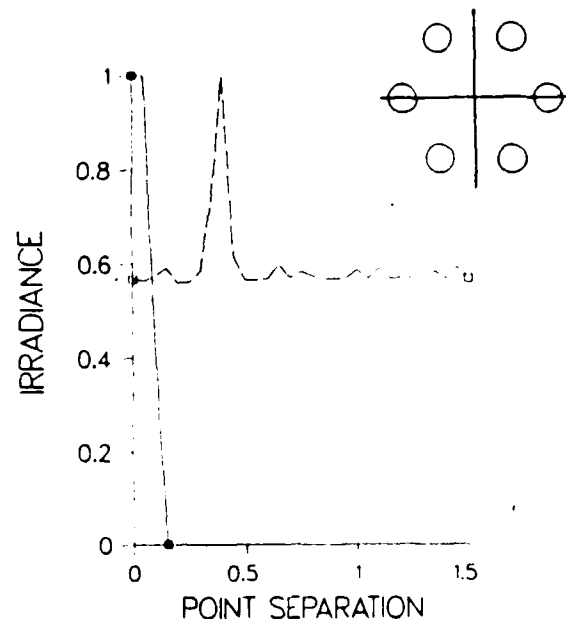
**a**



**b**



**c**



**d**

Fig. 3.18. Irradiance vs. Point Separation Plots for Incoherently Illuminated Six Aperture System with Aperture-Origin Separations of a)  $2.00a$ , b)  $3.00a$ , c)  $4.00a$ , and d)  $5.00a$  (— Central Value, — — — Side Lobe Maxima).

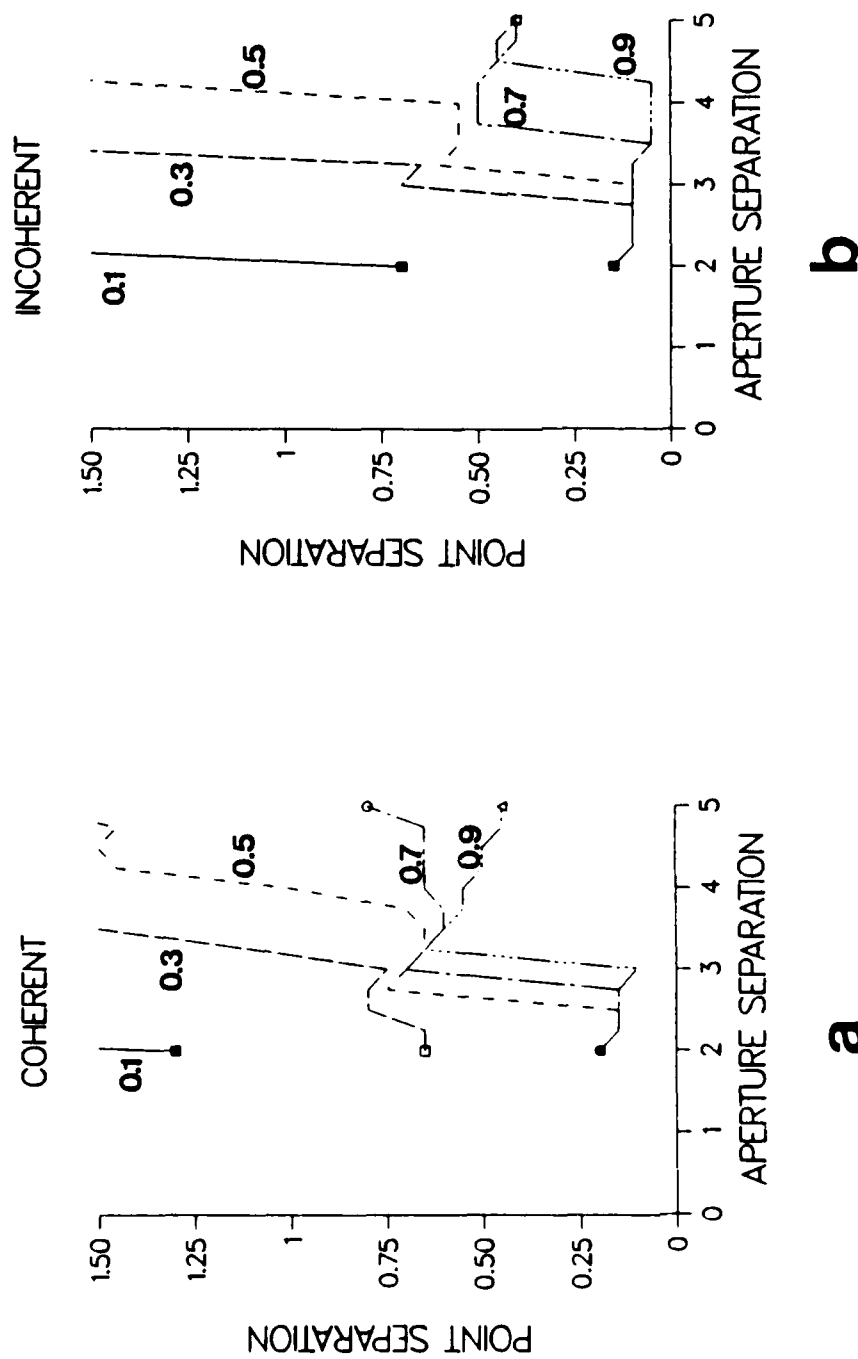


Fig. 3.19. Threshold Plots for Six Aperture System  
 Illuminated a) Coherently and b) Incoherently  
 (Threshold Values: 0.1, 0.3, 0.5, 0.7, 0.9).

four aperture cases, these plots allow one to evaluate and compare the two-point resolution performance of a six aperture system illuminated coherently and incoherently. As in the three and four aperture systems, the point separation required to achieve two-point resolution at all thresholds, at a given aperture-origin separation, in the incoherent case was consistently less than the coherent case. This is particularly evident at a threshold of 0.9 and an aperture-origin separation of  $4.00a$ . In this configuration, the coherently illuminated system required a 0.60 point separation for two-point resolution, whereas the incoherently illuminated system required only a 0.10 point separation to resolve the two point sources.

#### Six Aperture System - Point Sources Rotated $20^\circ$

The configuration for imaging the two point sources rotated  $20^\circ$  in the  $x_0$ - $y_0$  axis of the object plane is depicted in Figure 3.20. As the point sources were rotated in the object plane, there was a change in the point separations necessary to insure two-point resolution. This was a result of the side lobes of each diffraction pattern rotating about their respective central lobes. As a result, the summed fields (coherent illumination) or irradiances (incoherent illumination) of the side lobes differed from those of the system where the two point sources were located on the  $x_0$  axis in the object plane. To illustrate this point, refer to

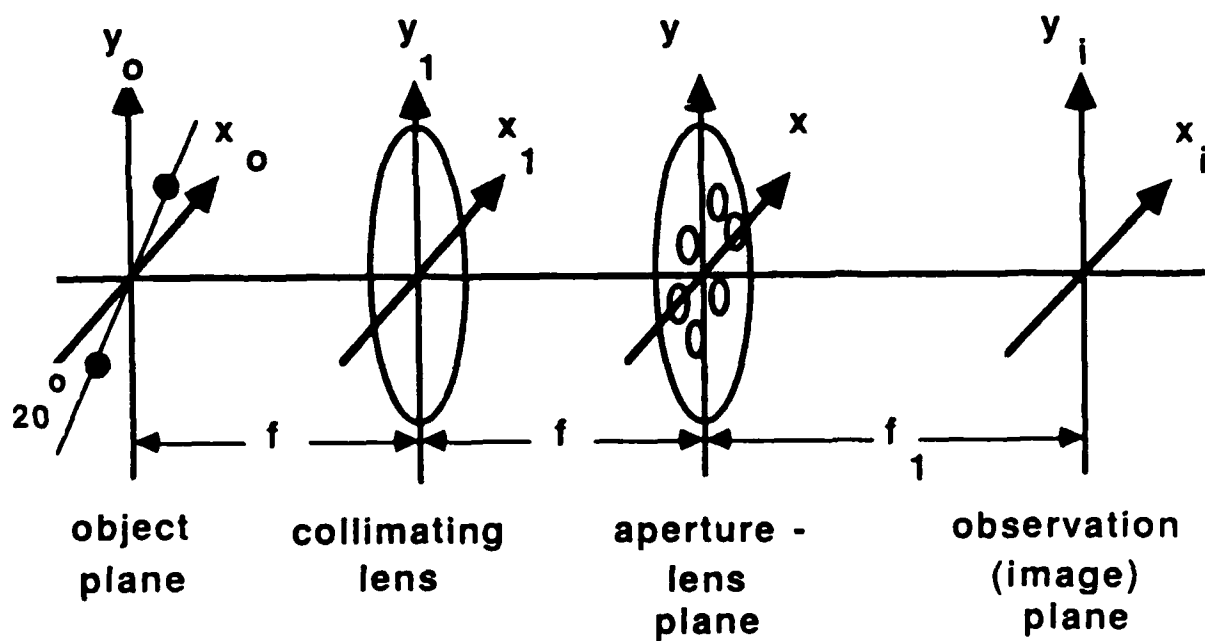


Fig. 3.20. Configuration for Observing Irradiance Patterns from Two Point Sources Rotated  $20^\circ$  in the  $X_0$ - $Y_0$  Plane.

Figures 3.21a and b. Figure 3.21a represents the two-point coherent illumination of an six aperture system, with aperture-origin separation of  $2.00a$ , where the two point sources were on the  $x_0$  axis and had a point separation of  $0.5$ . Figure 3.21b represents the same configuration, however, the two point sources were rotated  $20^\circ$  in the  $x_0$ - $y_0$  plane. Because of the rotation, the resulting far-field diffraction pattern was altered.

Figures 3.22a through d illustrate the Irradiance vs. Point Separation plots for coherent illumination of two point sources which were rotated  $20^\circ$  in the  $x_0$ - $y_0$  plane. The aperture-origin separations were  $2.00a$ ,  $3.00a$ ,  $4.00a$ , and  $5.00a$ . Comparison with the same system with the two point sources located on the  $x_0$  axis (Figures 3.17a through d) indicated that the rotation of the two point sources produced a substantial change in the side lobe maxima at the various point separations. This was further emphasized when comparing the threshold plots in Figures 3.23a and b. The differences between the two plots is most evident at the  $0.7$  and  $0.9$  thresholds. From  $2.75a$  through  $4.00a$  aperture-origin separations, the system with the rotated point sources had superior two-point resolution performance at the  $0.7$  threshold. At the  $0.9$  threshold and  $3.00a$  to  $5.00a$  aperture-origin separation, this system clearly outperformed the system with the point sources on the  $x_0$  axis.



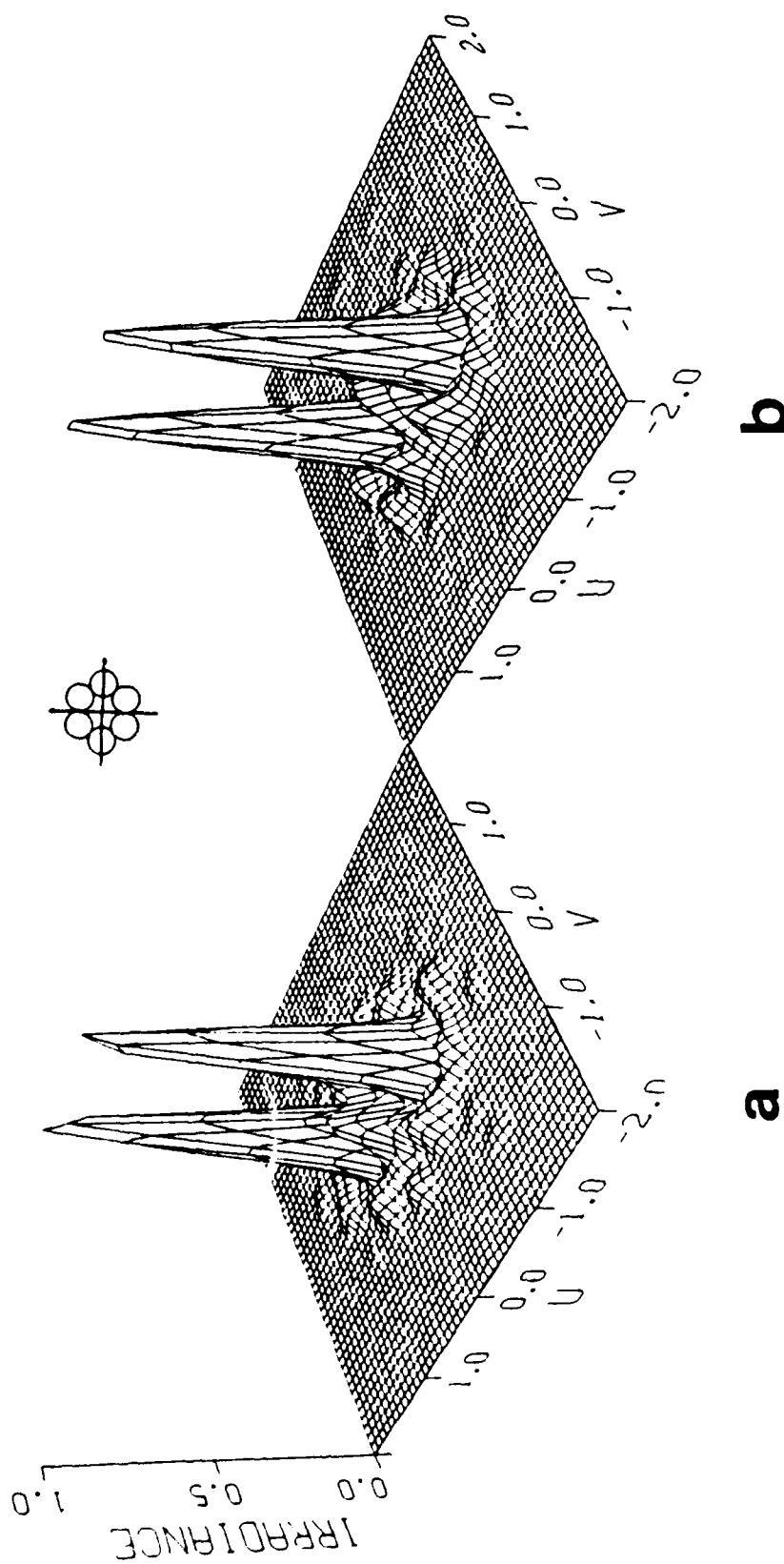
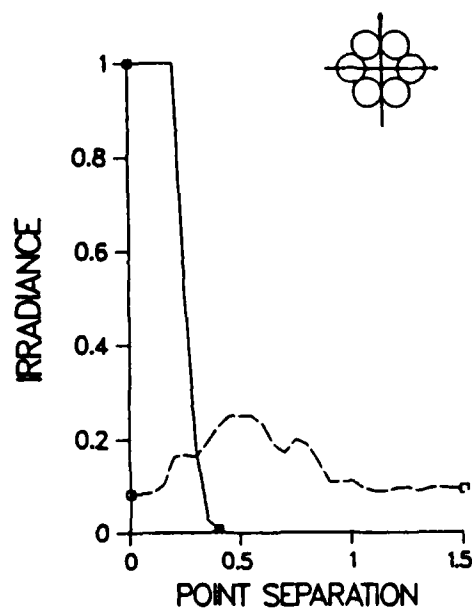
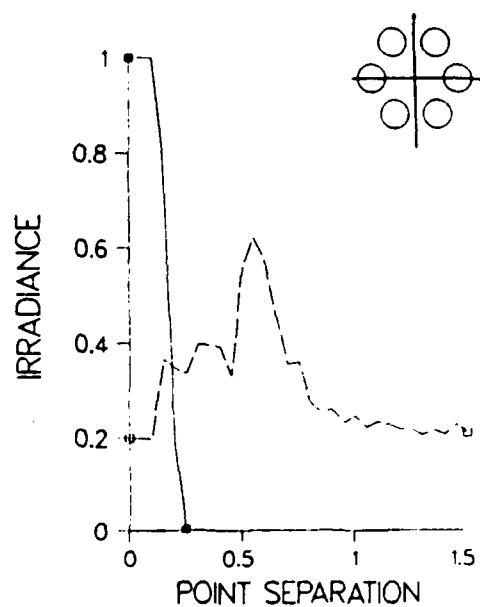


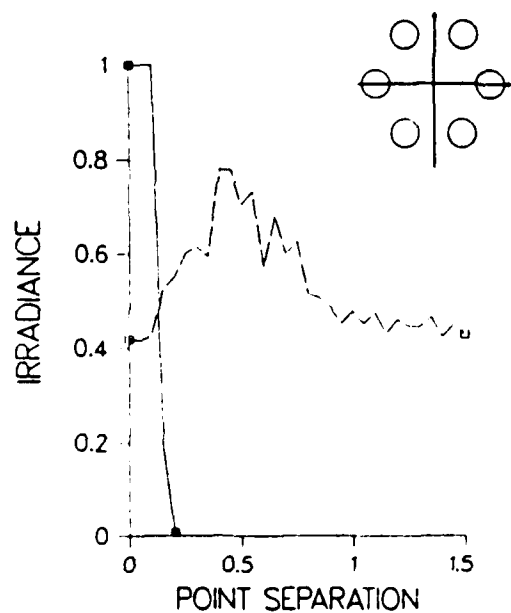
Fig. 3.21. Irradiance Patterns of a Six Aperture System with Aperture-Origin Separation of 2.00a  
Coherently Illuminated by Two Point Sources Separated by Normalized Distance of 0.50 on the a)  $X_0$  (Object Plane) Axis and b) Rotated  $20^\circ$  in the  $X_0 - Y_0$  (Object) Plane.



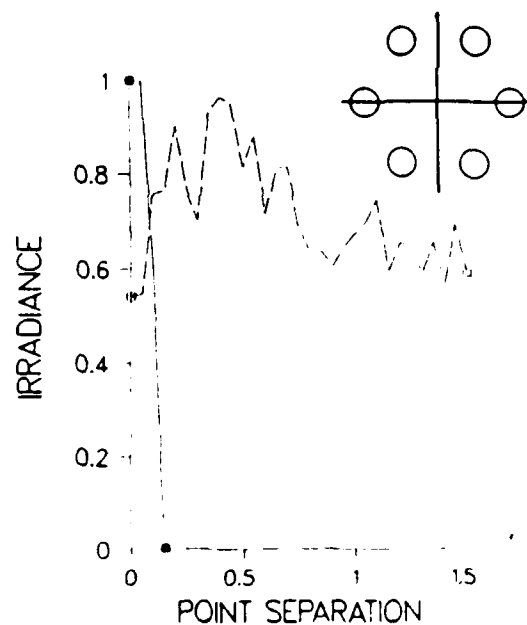
**a**



**b**



**c**



**d**

Fig. 3.22. Irradiance vs. Point Separation Plots for Coherently Illuminated, Six Aperture System with Aperture-Origin Separations of a)  $2.00a$ , b)  $3.00a$ , c)  $4.00a$ , and d)  $5.00a$  with the Two Point Sources Rotated  $20^\circ$  in the  $X_o - Y_o$  (Object) Plane.

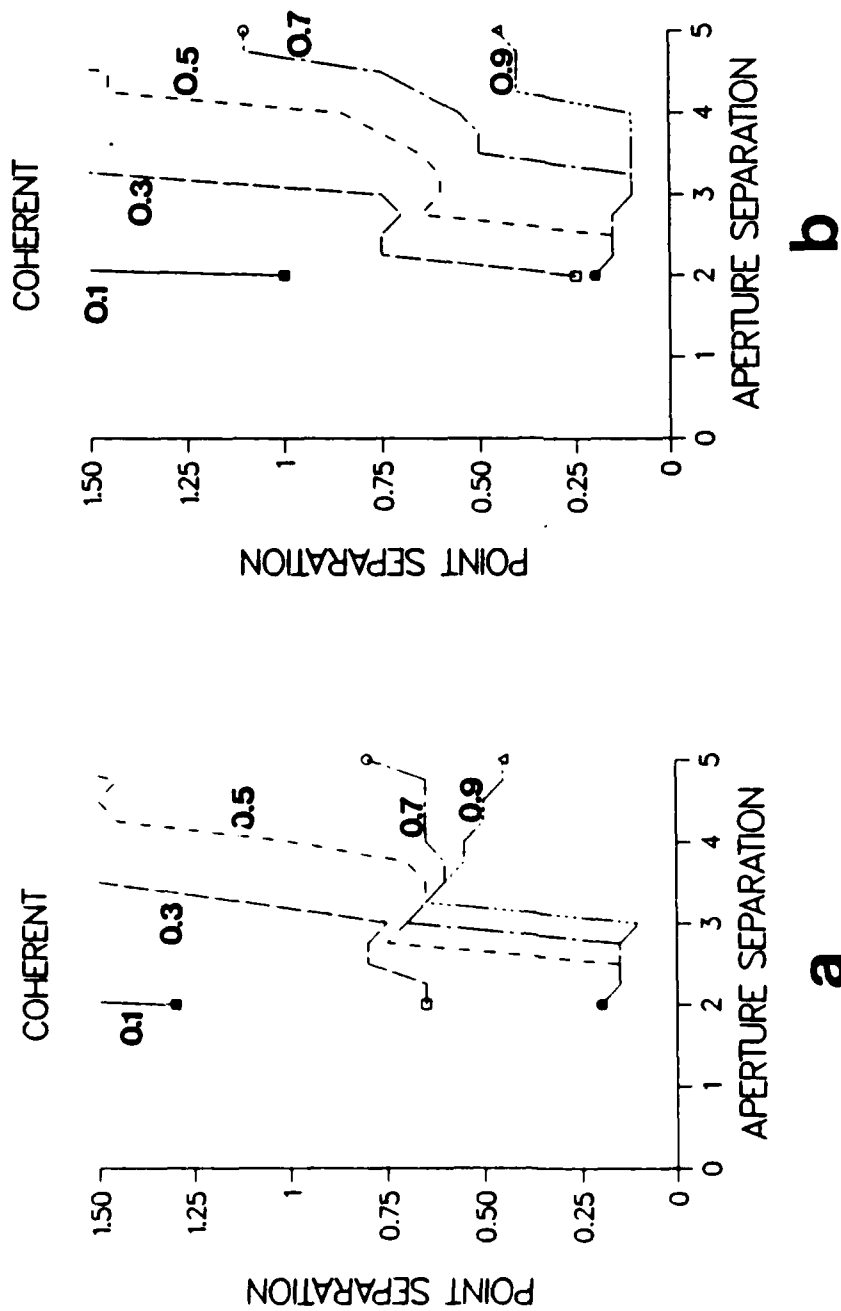


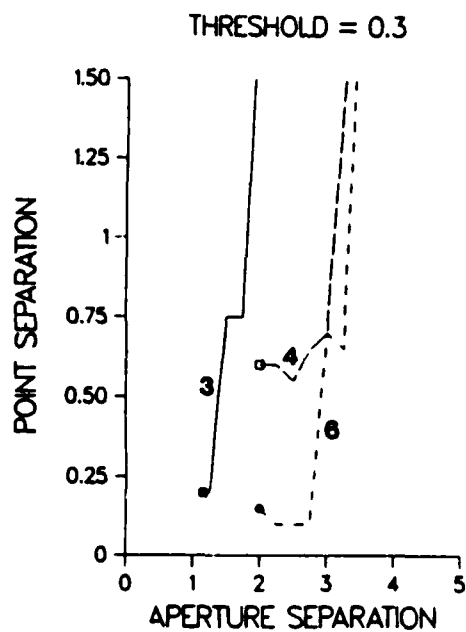
Fig. 3.23. Threshold Plots for Six Aperture System Illuminated Coherently with Two Point Sources  
 a) Located on the  $X_0$  Axis and b) Rotated  $20^\circ$  in the  $X_0-Y_0$  (Object) Plane (Threshold Values:  $\frac{0.1}{0.5}$ ,  $\frac{0.3}{0.7}$ ,  $\frac{0.5}{0.9}$ ).

## Multi-Aperture Systems Comparison

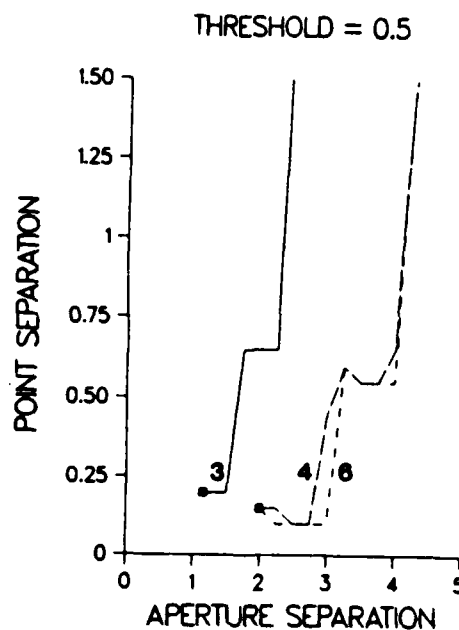
The threshold plots provided the information necessary to choose among several multi-aperture optical systems for superior two-point resolution performance. To demonstrate the use of the threshold plots in comparing the two-point resolution performance of the three, four, and six aperture systems, refer to Figures 3.24a through d. These curves represent the 0.3, 0.5, 0.7, and 0.9 threshold values for each multi-aperture system illuminated incoherently.

Figure 3.24a represents the two-point resolution performance of the three multi-aperture systems at a threshold value of 0.3. It is evident from this plot that the six aperture system outperforms the three and four aperture systems except at an aperture-origin separation of  $3.00a$ . At this aperture-origin separation, the four and six aperture threshold curves intersect which indicates that the two systems were able to initially resolve the two point sources at a point separation of 0.70.

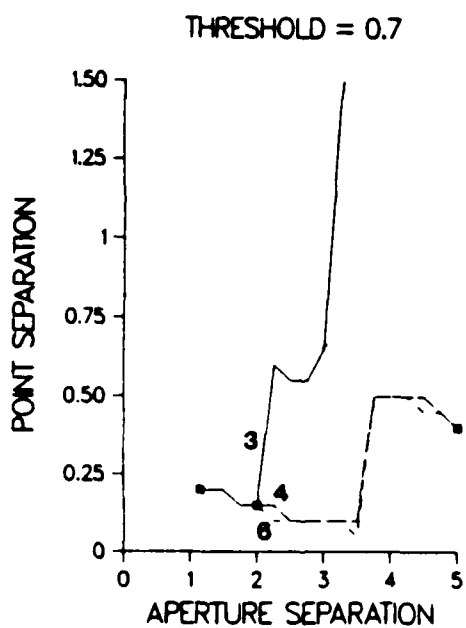
As the threshold values rose to the 0.9 value (Figures 3.24b through d), the two-point performance of the four and six aperture systems became more closely aligned. This is most pronounced in Figure 3.24d. At the 0.9 threshold value, the threshold curves of the four and six aperture systems are identical except at the aperture-origin separations of  $2.00a$  to  $2.50a$  and  $3.25a$  to  $3.75a$ . At these separations, the six aperture system slightly outperformed the four aperture system.



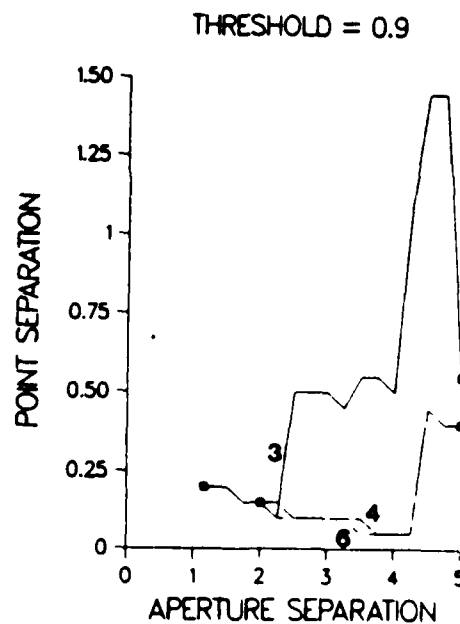
**a**



**b**



**c**



**d**

Fig. 3.24. Threshold Plots for Comparing the Incoherent Two-Point Resolution Performance of the Three (—), Four (— — —), and Six (— · —) Aperture Systems at Threshold Values of a) 0.3, b) 0.5, c) 0.7, and d) 0.9.

At all threshold values, except at the aperture-origin separation of  $2.00a$ , the four and six aperture systems outperformed the three aperture system. At an aperture-origin separation of  $2.00a$  and  $0.7$  and  $0.9$  threshold values, all three aperture systems performed equally well and were able to resolve two point sources at a point separation of  $0.14$ .

#### IV. Conclusions

This research has proposed a new two-point resolution criterion for multi-aperture optical systems for both coherent and incoherent illumination. This criterion is based on the use of thresholds of the resulting far-field diffraction patterns of multi-aperture optical systems. The thresholds, which are defined as a fraction of the central lobe irradiance of the imaged point, varied from 0.1 to 0.9 of the central lobe irradiance of the far-field diffraction patterns. Any lobe irradiance which was greater than the specified threshold was observed while those with irradiances less than the specified threshold were disregarded. When the side lobe irradiance maxima were less than the threshold value and the two-point separation was greater than the Sparrow limit, the two point sources were considered resolved.

Additional conclusions, which resulted from the analysis of the three different multi-aperture systems using the threshold criterion, follow:

- (1) The threshold plots provided the information necessary to compare the two-point resolution performance of a particular multi-aperture optical system illuminated coherently and incoherently.

- (2) The threshold plots provided the information necessary to compare the two-point resolution performance of different multi-aperture systems for incoherent illumination.
- (3) The two-point resolution performance for each multi-aperture system incoherently illuminated was superior to that coherently illuminated.
- (4) The point separation required to satisfy the Sparrow like criterion decreased as the aperture-origin separation increased for both coherent and incoherent illumination for the three multi-aperture systems evaluated.
- (5) Rotation of the two point sources in the object plane produced a substantial change in the side lobe maxima at various point separations.



Appendix: ComputerCodes



```

C  PARAMETER DEGREE = THE ANGLE IN DEGREES, TO DESCRIBE THE LOCATION OF THE
C  CENTER OF EACH APERTURE. DEGREE IS ENTERED AS A
C  REAL NUMBER.
C  PARAMETER DIS = DISTANCE OF EACH APERTURE FROM THE ORIGIN. THIS
C  IS EXPRESSED AS A FRACTION OF THE RADIUS OF EACH
C  APERTURE. WHEN DIS = 2.0 FOR THE SYMMETRIC HEX
C  APERTURE SYSTEM, THE APERTURES ARE JUST TOUCHING
C  EACH OTHER. IN THIS CONFIGURATION, ONE OF THE
C  SUBAPERTURES COULD BE PLACED IN THE CENTER OF
C  SYSTEM SO THAT IT WAS JUST TOUCHING THE SIX
C  SURROUNDING SUBAPERTURES.
C  PARAMETER UNRM = NORMALIZATION FACTOR FOR THE FINAL
C  OUTPUT OF THE INTENSITY
C
38  FORMAT(F10.6,3X,I3)
39  FORMAT(I3)
40  FORMAT(F10.6)
41  FORMAT(I3,3X,I3,3X,F10.4,3X,F10.4)
42  FORMAT(I3,3X,I3,3X,F10.4)
44  FORMAT(I3,3X,F10.4,3X,F10.4)
50  FORMAT(' PERFORMING COHERENT IMAGING')
51  FORMAT(' PERFORMING INCOHERENT IMAGING')
55  FORMAT(' VALUE OF SECONDARY SIDE LOBE IN R.H.S. PLANE IS ')
60  FORMAT(' VALUE OF SECONDARY SIDE LOBE IN L.H.S. PLANE IS ')
61  FORMAT(' MAX SECONDARY PEAK IS GREATER THAN PRIMARY
62  & PEAK ON R.H.S. OF PLANE')
62  FORMAT(' MAX SECONDARY PEAK IS GREATER THAN PRIMARY
63  & PEAK ON L.H.S. OF PLANE')
63  FORMAT(' MAX SECONDARY VALUE IS AT (24,24)')
64  FORMAT(' POINT SEPARATION = ',F10.6)
65  FORMAT(' INTENSITY AT (24,24) = ',F10.6)
66  FORMAT(' R.H.S. PRIMARY PEAK VALUE = ',F10.6)
67  FORMAT(' L.H.S. PRIMARY PEAK VALUE = ',F10.6)
C
C  PI=3.1415927
C  IF (COHER.EQ.50)WRITE(*,50)
C  IF (COHER.EQ.51)WRITE(*,51)
C
C  COMPUTE THE PHASE TERM FOR THE PHASE ANNULUS
C
C  TA = CEXP(CMPLX(0.0,PHASE))
C  IF (PHASE.EQ.0.00)TA = 0.00
C
C  READ IN VALUES OF THE LOCATION OF THE APERTURES
C
C  DO 105 I1=1,NM
C  READ(5,40)DEGREE
C  IX(I1)=COS(DEGREE*PI/180.0)
C  IY(I1)=SIN(DEGREE*PI/180.0)
C  READ(5,40)DIS
105  DIST(I1)=DIS
C
C  INPUT VALUES OF POINT SEPARATION AND INITIALIZE CENTMIN
C
C  I100 = 0
C  CENTMIN=40
C
C  COMMENT OUT NEXT LINE IF NOT FINDING 2-DIA INTENSITY VS.
C  POINT SEPARATION PLOTS
C
C  DO 1000 PTSEP=0.0,1.55,.05
C  I100 = I100 + 1
C  PTSEP=0.7
C  WRITE(*,64)PTSEP

```

```

      DO 275 I=1,48
      DO 280 J=1,48
        Y=(J - 24.0)*.275
        X=(1 + (24.0 - 2.0 * I))* .275
        IF (X.EQ.0.00000000)X=0.000001
C
C  COMPUTE ARGUMENT FOR FIRST BESSEL FUNCTION (ENVELOPE FUNCTION)
C
      Z=SQRT((X-PI*PTSEP)**2.0 + Y**2.0)
C
C  CALL UP MMBSJ1 (I*SL) TO COMPUTE THE BESSEL FUNCTION
C  DUE TO ORIGINAL UNOBSTRUCTED APERTURE FOR POINT SOURCE AT -PTSEP.
C
      ARG = Z
      CALL MMBSJ1 (Z,IER)
      E = MMBSJ1(Z,IER)
      E = E/Z
      IF (ANNRAD.EQ.0.00)GO TO 200
C
C  CALL UP MMBSJ1 TO COMPUTE BESSEL FUNCTION FOR THE
C  OBSTRUCTING APERTURE FOR POINT SOURCE AT -PTSEP.
C
      Z1 = ANNRAD*Z
      ARG = Z1
      CALL MMBSJ1 (ARG,IER)
      E1 = MMBSJ1 (ARG,IER)
      E1 = ((ANNRAD)**2)*E1/Z1
C
C  COMPUTE FUNCTIONS DUE TO LOCATION AND SPACING OF
C  THE 6 APERTURES FOR POINT SOURCE AT -PTSEP.
C
200      S=CMPLX(0.0,0.0)
      DO 100 I1=1,404
        X1 = -((X*MX(I1)*DIST(I1)) - (MX(I1)*PI*DIST(I1)*PTSEP))
        Y1 = -Y*MY(I1)*DIST(I1)
        G(I1)=CEXP(CMPLX(0.0,X1 + Y1))
100      S = S + G(I1)
      S2=CABS(S)
C
C  THE FIELD AMPLITUDE DUE TO THE POINT SOURCE LOCATED AT -PTSEP IS:
C
      CONTINUE
      IF (ANNRAD.EQ.0.00)E1=0.00
      A(I,J) = (E-(E1*(1.00-TA)))* S2
      IF (COHER.EQ.5)GO TO 700
C
C  THE INTENSITY DUE TO THE POINT SOURCE LOCATED AT -PTSEP IS:
C  (THIS NEXT LINE IS USED FOR INCOHERENT IMAGING ONLY.
C
      A(I,J) = A(I,J)**2
700      CONTINUE
      IF (PTSEP.EQ.0.00) GO TO 28
C
C  COMPUTE THE ARGUMENT FOR THE SECOND BESSEL FUNCTION (ENVELOPE FUNCTION)
C
26      P = SQRT((X+PI*PTSEP)**2.0 + Y**2.0)
C
C  CALL UP MMBSJ1 (I*SL) TO COMPUTE BESSEL FUNCTION DUE TO
C  ORIGINAL UNOBSTRUCTED APERTURE FOR POINT SOURCE AT +PTSEP.
C
      ARG = P
      CALL MMBSJ1 (ARG,IER)
      F = MMBSJ1(ARG,IER)
      F = F/P
      IF (ANNRAD.EQ.0.00)GO TO 3

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```

C
C CALL UP MMBSJ1 TO COMPUTE BESSEL FUNCTION FOR THE
C SUBSTRUCTING APERTURE FOR POINT SOURCE AT +PTSEP.
C
P1 = ANNRAD * P
ARG = P1
CALL MMBSJ1 (ARG,IER)
F1 = MMBSJ1 (ARG,IER)
F1 = ((ANNRAD**2)*F1)/P1
C
C COMPUTE FUNCTION DUE TO THE LOCATION AND SPACING OF THE
C SIX APERTURES FOR POINT SOURCE AT +PTSEP.
C
3 V = CMPLX(0.0,0.0)
DO 125 I1=1,NM1
X1=-((X*NX(I1)*DIST(I1)) + (NX(I1)*PI*DIST(I1)*PTSEP))
Y1 = -Y*NY(I1)*DIST(I1)
G(I1) = CEXP(CMPLX(0.0,X1 + Y1))
125 V = V + G(I1)
V1=CABS(V)
C
C THE FIELD AMPLITUDE DUE TO THE POINT SOURCE LOCATED AT +PTSEP IS:
C
CONTINUE
IF (ANNRAD.EQ.0.00)F1=0.00
B(I,J) = (F-(F1*(1.00-FA))) * V1
IF (COHER.EQ.50)GO TO 701
C
C THE INTENSITY DUE TO THE POINT SOURCE LOCATED AT +PTSEP IS:
C (USE THE NEXT LINE FOR INCOHERENT IMAGING ONLY)
C
B(I,J) = B(I,J)**2
701 CONTINUE
C
C THE TOTAL INTENSITY DUE TO BOTH POINT SOURCES IS (FOR INCOHERENT):
C THE TOTAL FIELD AMPLITUDE DUE TO BOTH POINT SOURCES IS (FOR COHERENT):
C
28 IF (PTSEP.EQ.0.00)B(I,J)=0.00
C
C IF NEED TO LOOK AT FIELD FROM A(I,J) ONLY, TAKE COMMENT OFF OF NEXT
C LINE AND PLACE COMMENT ON A(I,J)=0.0
C
B(I,J)=0.0
C
C IF NEED TO LOOK AT FIELD FROM B(I,J) ONLY, TAKE COMMENT OFF OF NEXT
C LINE AND PLACE COMMENT ON B(I,J)=0.0
C
A(I,J)=0.0
C(I,J) = A(I,J) + B(I,J)
IF (COHER.EQ.50)GO TO 702
C
C THE INTENSITY IN THE IMAGE PLANE IS (USE FOR INCOHERENT):
C
INT(I,J)=C(I,J)/DNORM
IF (COHER.EQ.50)GO TO 290
C
C USE THE NEXT LINE FOR COHERENT IMAGING ONLY. THIS LINE WILL PROVIDE
C INTENSITY INFORMATION
C
702 INT(I,J)=C(I,J)**2/DNORM
C
C IF NEED TO LOOK AT FIELD AMPLITUDE, PLACE COMMENT ON LINE ABOVE AND TAKE
C COMMENT OFF OF THE NEXT-LINE
C
702 INT(I,J)=C(I,J)/DNORM

```

```

C
280 CONTINUE
275 CONTINUE
C
C FIND THE MAXIMUM VALUE OF INTENSITY
C
      H=INT(1,1)
      DO 10 I=1,48
      DO 20 J=1,48
      IF (INT(I,J).GT.H)H=INT(I,J)
20 CONTINUE
10 CONTINUE
C
C NORMALIZE INTENSITY TO A MAXIMUM VALUE OF 1.00
C
      DO 15 I=1,48
      DO 17 J=1,48
      INT(I,J)=INT(I,J)/H
C
C NEXT LINE SETS VALUES FOR THRESHOLD VALUES OF INTENSITY
C
C      IF (INT(I,J).LT..5)INT(I,J)=.5
C
C COMMENT OUT NEXT LINE IF FINDING 2 DIA INTENSITY VS.
C POINT SEPARATION PLOTS
C
C      WRITE(15,42)I,J,INT(I,J)
17 CONTINUE
15 CONTINUE
C
C FIND THE MAX INTENSITY ON R.H.S. OF PLANE
C
      H1=INT(1,1)
      DO 11 I=1,24
      DO 21 J=1,48
      IF (INT(I,J).GT.H1)H1=INT(I,J)
      IF (H1.EQ.INT(I,J))K1=I
      IF (H1.EQ.INT(I,J))L1=J
21 CONTINUE
11 CONTINUE
C
C FIND THE POINT SEPARATION WHEN THE SUMMATION OF THE TWO
C PRIMARY PEAKS REACHES A MINIMUM. THIS NEXT PROCEDURE PERFORMS
C THIS FUNCTION. CENTMIN IS THE VALUE OF 1100 (POINT SEPARATION)
C WHERE THE SUMMATION OF THE TWO PRIMARY PEAKS IS A MIN.
C
      CENT(1100)=INT(24,24)
      IF (CENT(1100).GT..99.(CENT(1100).GT.CENTMIN))GO TO 1271
      DO 1270 I=1,1100
      IF (CENT(I).GT.CENT(1100-1))CENTMIN=I-1
1270 CONTINUE
1281 CONTINUE
C
C CHECK IF HAVE A MAX INTENSITY (MAX SECONDARY INTENSITY)
C AT INT(24,24) WHEN THE PRIMARY PEAKS HAVE CEASED TO
C ADD TO ONE ANOTHER.
C
      IF ((1100.GT.CENTMIN).AND.(INT(24,24).GT..999))GO TO 1305
C
C NEXT LINE WILL EXECUTE IF SECONDARY MAX PEAK (NOT
C INCLUDING THE POSSIBLE MAX SECONDARY PEAK AT INT(24,24))
C > PRIMARY PEAK
C
      IF (L1.NE.24)GO TO 1305
      PRIM(1100)=H1

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      GU TO 1100
C
C FIND MAX OF PRIMARY PEAK ON R.H.S. OF PLANE WHEN SECONDARY
C PEAK (NOT INCLUDING A MAX SECONDARY PEAK LOCATED AT INT(24,24))
C IS GREATER THAN PRIMARY PEAK
C
1050 H50=INT(1,24)
      DO 1051 I=1,24
      IF (INT(1,24).GT.H50) H50=INT(1,24)
1051 CONTINUE
      PRIM(1100)=H50
      WRITE(*,61)
      WRITE(*,66) H50
C
C1100 IF ((K1.EQ.24).AND.(L1.EQ.24).AND.(PTSEP.GT.0.0)) WRITE(*,61)
1100 CONTINUE
C
C FIND THE MAX INTENSITY ON L.H.S. OF PLANE
C
      H2=INT(1,1)
      DO 12 I=24,48
      DO 22 J=1,48
      IF (INT(1,J).GT.H2) H2=INT(1,J)
      IF (H2.EQ.INT(1,J)) K2=J
      IF (H2.EQ.INT(1,J)) L2=J
C22 CONTINUE
C12 CONTINUE
      IF (L2.NE.24) GO TO 1150
      GO TO 1200
C
C FIND MAX OF PRIMARY PEAK ON L.H.S. OF PLANE WHEN THE
C SECONDARY PEAK IS GREATER THAN THE PRIMARY PEAK
C
C1150 H51=INT(48,24)
      DO 1151 I=48,24,-1
      IF (INT(1,24).GT.H51) H51=INT(1,24)
C1151 CONTINUE
      WRITE(*,62)
      WRITE(*,67) H51
C
C1200 CONTINUE
      IF ((K2.EQ.24).AND.(L2.EQ.24).AND.(PTSEP.GT.0.0)) WRITE(*,62)
C
1205 CONTINUE
C
C FIND THE LOCATION OF THE FIRST MIN TO THE RIGHT OF THE
C PRIMARY PEAK IN THE RIGHT PLANE
C
      DO 401 I3=K1,1,-1
      IF (INT(I3,L1).GT.INT(I3,L1)) GO TO 402
C401 CONTINUE
C402 I4=I3-1
C
C I4=I3-1 IS THE VALUE OF THE MIN
C
      I4MIN1=INT(I3,L1)
C
C FIND THE LOCATION OF THE FIRST MIN ABOVE THE PRIMARY PEAK
C IN THE RIGHT HAND PLANE
C
      DO 403 J3=L1,48
      IF (INT(K1,J3)+1).GT.INT(K1,J3)) GO TO 404
C403 CONTINUE
C404 J4MIN2=J3
C
C FIND THE LOCATION OF THE FIRST MIN TO THE LEFT OF THE PRIMARY

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C   MAX IN THE RIGHT HAND PLANE
C
  DO 405 I3=K1,24
    IF (INT(I3+1,L1).GT.INT(I3,L1))GO TO 406
405  CONTINUE
406  IMIN3R=I3
C
C   FIND THE LOCATION OF THE FIRST MIN BELOW THE PRIMARY
C   MAX IN THE RIGHT HAND PLANE
C
  DO 407 J3=L1,1,-1
    IF (INT(K1,J3-1).GT.INT(K1,J3))GO TO 408
407  CONTINUE
408  JMIN4R=J3
C
C   FIND THE SECONDARY MAX INTENSITY IN THE R.H.S. OF PLANE
C
  H3=INT(1,1)
  DO 13 I=1,24
    DO 23 J=1,48
      I3=I
      J3=J
      IF ((I3.LT.IMIN3R).AND.(I3.GT.IMIN1R)).AND.
        & ((J3.LT.JMIN3R).AND.(J3.GT.JMIN4R))GO TO 23
      IF (INT(I3,J3).GT.H3)GO TO 1800
      GO TO 23
1800  H3=INT(I3,J3)
      IRSECMAX=I3
23    CONTINUE
13    CONTINUE
C
C   IF HAVE A MAX SECONDARY INTENSITY AT INT(24,24) WHEN POINT
C   SEPARATION IS SUCH THAT PRIMARY PEAKS ARE NOT SUMMING AT INT(24,24)
C   EXECUTE NEXT LINE
C
  IF((I100.GT.CENTMIN).AND.(INT(24,24).GE..999))GO TO 1310
C
C   IF THE POINT SEPARATION OF THE TWO POINTS IS SUCH THAT THE PRIMARY
C   PEAKS ARE SUMMING AT INT(24,24), EXECUTE THE NEXT LINE
C
  IF(I100.LE.CENTMIN)GO TO 1595
  GO TO 1598
1810  PRIM(I100)=H3
  C   WRITE(*,55)
  C   PRINT*,INT(24,24)
  SEC(I100)=INT(24,24)
  GO TO 1599
C
C   FIND SECONDARY MAX INTENSITY IN RHS PLANE IF 2 PRIMARY PEAKS
C   ARE SUMMING AT INT(24,24); I.E. THERE IS A CONTRIBUTION FROM
C   THE SUMMATION OF THE TWO PRIMARY PEAKS.
C
C   FIND FIRST MIN TO RIGHT OF THE PRIMARY PEAK IN RHS
C   PLANE IF THE TWO PRIMARY PEAKS ARE SUMMING
C
1595  CONTINUE
  IF((I100.LE.CENTMIN).AND.(INT(24,24).LT..999))IRSECMAX=K1
  IF((I100.LE.CENTMIN).AND.(INT(24,24).LT..999))PRIM(I100)=H1
  DO 1475 I3=IRSECMAX,1,-1
    IF (INT(I3-1,L1).GT.INT(I3,L1))GO TO 1476
1475  CONTINUE
1476  I2MIN1R=I3
  C   PRINT*,PRIM(I100)
C
C   FIND THE SECONDARY MAX INTENSITY IN RHS PLANE IF THERE IS A

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C CONTRIBUION DUE TO THE SUMMING OF THE PRIMARY PEAKS.
C
  H3=INT(1,1)
  DO 1700 I=1,24
  DO 1705 J=1,48
  I3=I
  J3=J
  IF(((I3.GT.I2MIN1R).AND.(I3.LE.24)).AND.
    & ((J3.LT.JMIN2R).AND.(J3.GT.JMIN4R)))GO TO 1705
  IF(INT(I3,J3).GT.H3)H3=INT(I3,J3)
1705 CONTINUE
1700 CONTINUE
C
1598 CONTINUE
C WRITE(*,55)
C PRINT*,H3
C SEC(I100)=H3
1599 CONTINUE
C
C FIND THE LOCATION OF THE FIRST MIN TO THE RIGHT OF THE PRIMARY
C PEAK IN THE LEFT HAND PLANE
C
C DO 410 I4=K2,24,-1
C IF (INT(I4-1,L2).GT.INT(I4,L2))GO TO 411
C410 CONTINUE
C411 IMIN1L=I4
C
C FIND THE LOCATION OF THE FIRST MIN ABOVE THE PRIMARY
C MAX IN THE LEFT HAND PLANE
C
C DO 412 J4=L2,48
C IF (INT(K2,J4+1).GT.INT(K2,J4))GO TO 413
C412 CONTINUE
C413 JMIN2L=J4
C
C FIND THE LOCATION OF THE FIRST MIN TO THE LEFT OF THE PRIMARY
C MAX IN THE LEFT HAND PLANE
C
C DO 414 I4=K2,48
C IF (INT(I4+1,L2).GT.INT(I4,L2))GO TO 415
C414 CONTINUE
C415 IMIN3L=I4
C
C FIND THE LOCATION OF THE FIRST MIN BELOW THE PRIMARY
C MAX IN THE LEFT HAND PLANE
C
C DO 416 J4=L2,1,-1
C IF (INT(K2,J4-1).GT.INT(K2,J4))GO TO 417
C416 CONTINUE
C417 JMIN4L=J4
C
C FIND THE SECONDARY MAX INTENSITY IN L.H. PLANE
C
C H4=INT(1,1)
C DO 14 I=24,48
C DO 24 J=1,48
C I4=I
C J4=J
C IF(((I4.LT.I4MIN3L).AND.(I4.GT.IMIN1L)).AND.
C & ((J4.LT.JMIN2L).AND.(J4.GT.JMIN4L)))GO TO 24
C IF (INT(I4,J4).GT.H4)H4=INT(I4,J4)
C24 CONTINUE
C14 CONTINUE
C WRITE(*,60)
C PRINT*,H4

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C      IF ((H3.EQ.INT(24,24)).AND.(H4.EQ.INT(24,24)))WRITE(*,53)
C      WRITE(*,65)INT(24,24)
C
C TAKE OFF COMMENT ON NEXT LINE IF WANT APERTURE SEPARATION VS
C POINT SEPARATION 2-DIM PLOTS
C
1000--- CONTINUE
C
C FIND THE LOCATION (POINT SEPARATION) WHERE THE SUMMATION OF THE
C TWO PRIMARY PEAKS DECREASES FROM 1.00
C
      DO 1300 I10=1,CENTMIN
      IF(CENT(I10).LT.1.0)GO TO 1301
1300--- CONTINUE
1301--- INIT=I10
C
C WRITE INT(24,24) VALUES FROM EACH POINT SEPARATION WHERE
C THE TWO PRIMARY PEAKS SUMMED TOGETHER
C
      DO 1325 I10=1,CENTMIN
      WRITE(18,44)I10,CENT(I10)
1325--- CONTINUE
C
C WRITE PRIMARY PEAK VALUES FROM EACH POINT SEPARATION TO A FILE
C STARTING FROM X LOCATION INIT
C
      DO 1400 I20=INIT,I100
      WRITE(19,44)I20,PRIM(I20)
1400--- CONTINUE
C
C WRITE SECONDARY MAX VALUE FROM EACH POINT SEPARATION TO A FILE
C
      DO 1450 I30=1,I100
      WRITE(20,44)I30,SEC(I30)
1450--- CONTINUE
C
C FIND POINT SEPARATION WHERE THRESHOLD VALUE IS MET OR EXCEEDED
C
      DO 1960 I4=I100,1,-1
      IF((SEC(I4).GE..1).OR.(CENT(I4).GE..1))GO TO 1965
1960--- CONTINUE
1965--- WRITE(21,38)DIS,I4
      DO 1970 I4=I100,1,-1
      IF((SEC(I4).GE..2).OR.(CENT(I4).GE..2))GO TO 1975
1970--- CONTINUE
1975--- WRITE(22,38)DIS,I4
      DO 1980 I4=I100,1,-1
      IF((SEC(I4).GE..3).OR.(CENT(I4).GE..3))GO TO 1985
1980--- CONTINUE
1985--- WRITE(23,38)DIS,I4
      DO 1990 I4=I100,1,-1
      IF((SEC(I4).GE..4).OR.(CENT(I4).GE..4))GO TO 1995
1990--- CONTINUE
1995--- WRITE(24,38)DIS,I4
      DO 2000 I4=I100,1,-1
      IF((SEC(I4).GE..5).OR.(CENT(I4).GE..5))GO TO 2005
2000--- CONTINUE
2005--- WRITE(25,38)DIS,I4
      DO 2010 I4=I100,1,-1
      IF((SEC(I4).GE..6).OR.(CENT(I4).GE..6))GO TO 2015
2010--- CONTINUE
2015--- WRITE(26,38)DIS,I4
      DO 2020 I4=I100,1,-1
      IF((SEC(I4).GE..7).OR.(CENT(I4).GE..7))GO TO 2025
2020--- CONTINUE

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2025  WRITE(27,38)DIS,I4
      DO 2030 I4=1100,1,-1
      IF((SEC(I4).GE..8).OR.(CENT(I4).GE..8))GO TO 2035
2030  CONTINUE
2035  WRITE(28,38)DIS,I4
      DO 2040 I4=1100,1,-1
      IF((SEC(I4).GE..9).OR.(CENT(I4).GE..9))GO TO 2045
2040  CONTINUE
2045  WRITE(29,38)DIS,I4
C  WRITE CENTRAL SLICE OF INTENSITY; I.E. A ONE DIMENSIONAL VIEW.
C  THE FIRST LOOP(J=1,49) VIEWS THE CENTRAL SLICE ALONG THE Y AXIS(X=0.0)
C
      DO 426 J=1,49
      WRITE(17,44)J,INT(24,J)
426  CONTINUE
C  THIS SLICE VIEWS ALONG THE X AXIS (Y=0.0)
C
      DO 425 I=1,49
      WRITE(16,44)I,INT(1,24)
425  CONTINUE
      CLOSE(UNIT=15)
      CLOSE(UNIT=16)
      CLOSE(UNIT=17)
      CLOSE(UNIT=18)
      CLOSE(UNIT=19)
      CLOSE(UNIT=20)
      CLOSE(UNIT=21)
      CLOSE(UNIT=22)
      CLOSE(UNIT=23)
      CLOSE(UNIT=24)
      CLOSE(UNIT=25)
      CLOSE(UNIT=26)
      CLOSE(UNIT=27)
      CLOSE(UNIT=28)
      CLOSE(UNIT=29)
      STOP
      END

```

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## VITA

Major Steven M. Watson was born on 24 November 1952 in the U.S. Army General hospital, Tokyo, Japan. He graduated from high school in 1970 and attended the U.S.A.F. Academy where he received the degree of Bachelor of Science in Geography and his commission in June 1975. He then attended Undergraduate Navigator Training and received his wings in April 1976. He served as a navigator on RC-135 aircraft at Eielson AFB, Alaska and Offutt AFB, Nebraska from 1976 through 1981. During this time, he received the degree of Master of Science in Physics at Creighton University, Omaha, Nebraska. During 1982, he attended the USAF Test Pilot School and graduated as an Experimental Flight Test Navigator. From 1983 through 1985, he was assigned to the 4950th Test Wing at Wright-Patterson AFB, Ohio until entering the School of Engineering, Air Force Institute of Technology, in June 1985.

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## REPORT DOCUMENTATION PAGE

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Two-point resolution criteria is the classic way of comparing telescopes. However, the standard two-point resolution criterion is not appropriate for multi-aperture systems. This paper proposes a new two-point resolution criterion based on the idea of thresholding the irradiances of the resulting far-field diffraction patterns of multi-aperture optical systems. The threshold was defined as a fraction of the central lobe irradiance. The thresholds varied from 0.1 to 0.9 of the central lobe irradiance of the far-field diffraction patterns. Theoretical data of normalized irradiance versus point separation for various multi-aperture optical systems were presented. The two-point resolution for these configurations was analyzed. The two-point resolution criterion using thresholds was demonstrated. The threshold criterion provided the information necessary to compare the two-point resolution performance of a particular multi-aperture optical system illuminated coherently and incoherently. Also, this criterion allowed the comparison of the two-point resolution performance of systems composed of three, four, and six subapertures illuminated incoherently. (TOP SECRET)

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